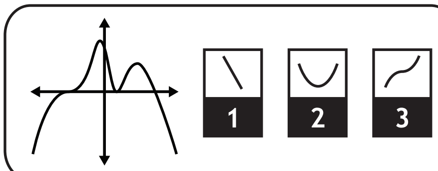
Mathematics 30-1

Student Workbook

Unit 1



Lesson 1: Polynomial Functions
Approximate Completion Time: 3 Days

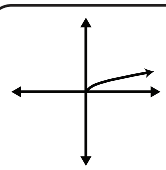
1	3	-4	-5	2
-	↓	3	-7	2
	3	-7	2	0

Lesson 2: Polynomial Division
Approximate Completion Time: 3 Days

$x^3 - 5x^2 + 2x + 8$

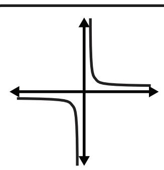
$(x + 1)(x - 2)(x - 4)$

Lesson 3: Polynomial Factoring
Approximate Completion Time: 3 Days



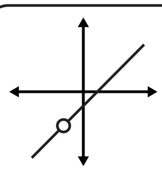
$y = \sqrt{x}$

Lesson 4: Radical Functions
Approximate Completion Time: 2 Days



$y = \frac{1}{x}$

Lesson 5: Rational Functions I
Approximate Completion Time: 2 Days



$y = \frac{x^2 + x - 2}{x + 2}$

Lesson 6: Rational Functions II
Approximate Completion Time: 3 Days

UNIT ONE
Polynomial, Radical and Rational Functions

Mathematics 30-1

Student Workbook

Unit 1

Complete this workbook by watching the videos on **www.math30.ca**.
Work neatly and use proper mathematical form in your notes.

UNIT ONE

Polynomial, Radical and Rational Functions

Lesson Notes

Example 1

Introduction to Polynomial Functions.

Defining Polynomials

a) Given the general form of a polynomial function:

$$P(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_1x^1 + a_0$$

the leading coefficient is ______.

the degree of the polynomial is ______.

the constant term of the polynomial is ______.

For each polynomial function given below, state the leading coefficient, degree, and constant term.

i) $f(x) = 3x - 2$

leading coefficient: ______ degree: ______ constant term: ______

ii) $y = x^3 + 2x^2 - x - 1$

leading coefficient: ______ degree: ______ constant term: ______

iii) $P(x) = 5$

leading coefficient: ______ degree: ______ constant term: ______

b) Determine which expressions are polynomials. Explain your reasoning.

i) $x^5 + 3$

polynomial: yes no

ii) $5^x + 3$

polynomial: yes no

iii) 3

polynomial: yes no

iv) $4x^2 - 5x^{\frac{1}{2}} - 1$

polynomial: yes no

v) $x^2 + \frac{1}{3}x - 4$

polynomial: yes no

vi) $|x|$

polynomial: yes no

vii) $5\sqrt{x} - 1$

polynomial: yes no

viii) $\sqrt{7}x + 2$

polynomial: yes no

ix) $\frac{1}{x + 3}$

polynomial: yes no

Polynomial, Radical, and Rational Functions
LESSON ONE - *Polynomial Functions*
Lesson Notes

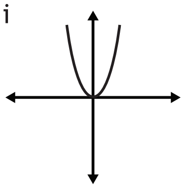
Example 2

End Behaviour of Polynomial Functions.

Even-Degree Polynomials

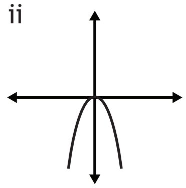
a) The equations and graphs of several even-degree polynomials are shown below. Study these graphs and generalize the end behaviour of even-degree polynomials.

i



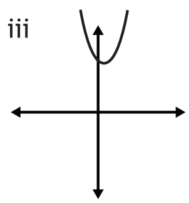
$f(x) = x^2$
quadratic

ii



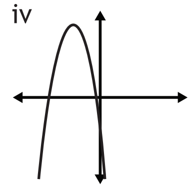
$f(x) = -x^2$
quadratic

iii



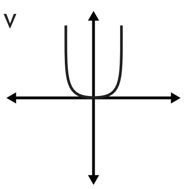
$f(x) = x^2 - x + 6$
quadratic

iv



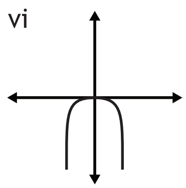
$f(x) = -x^2 - 8x - 7$
quadratic

v



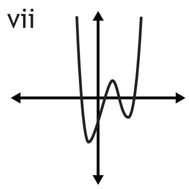
$f(x) = x^4$
quartic

vi



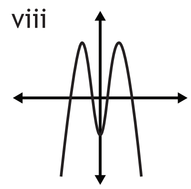
$f(x) = -x^4$
quartic

vii



$f(x) = x^4 - 4x^3 + x^2 + 7x - 3$
quartic

viii



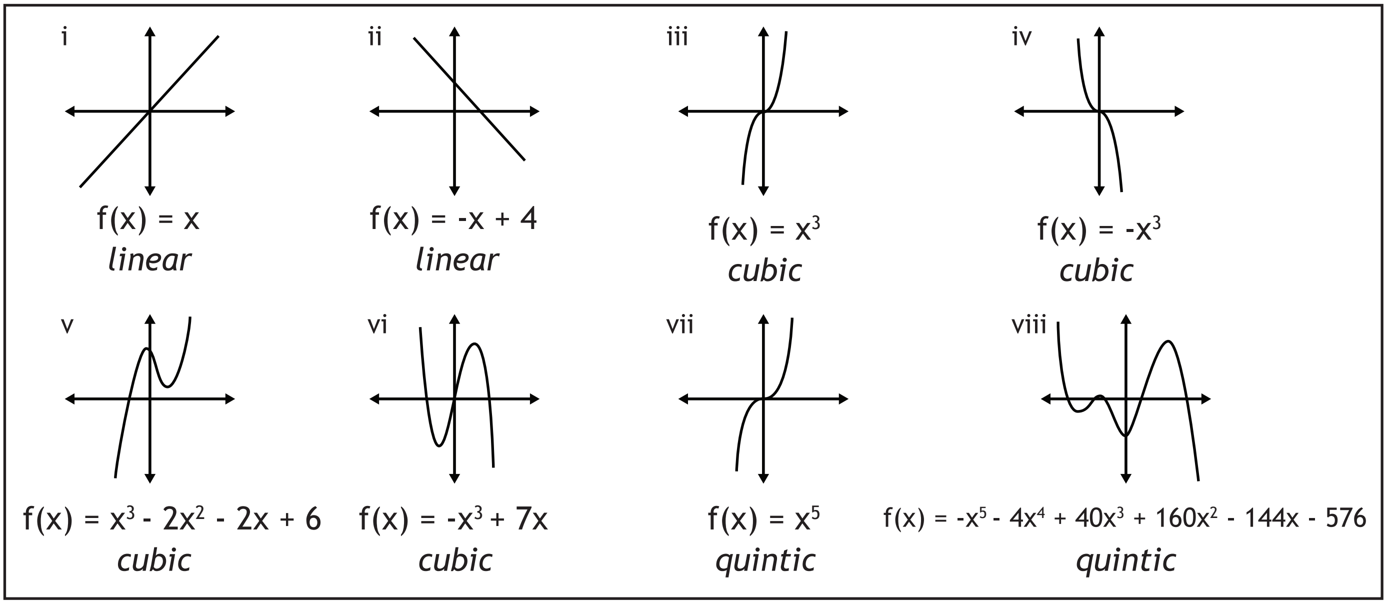
$f(x) = -x^4 + 7x^2 - 5$
quartic

End behaviour of even-degree polynomials:

Sign of Leading Coefficient	*End Behaviour*
Positive	
Negative	

b) The equations and graphs of several odd-degree polynomials are shown below. Study these graphs and generalize the end behaviour of odd-degree polynomials.

Odd-Degree Polynomials



End behaviour of odd-degree polynomials:

Sign of Leading Coefficient	*End Behaviour*
Positive	
Negative	

Example 3

Zeros, Roots, and x-intercepts of a Polynomial Function.

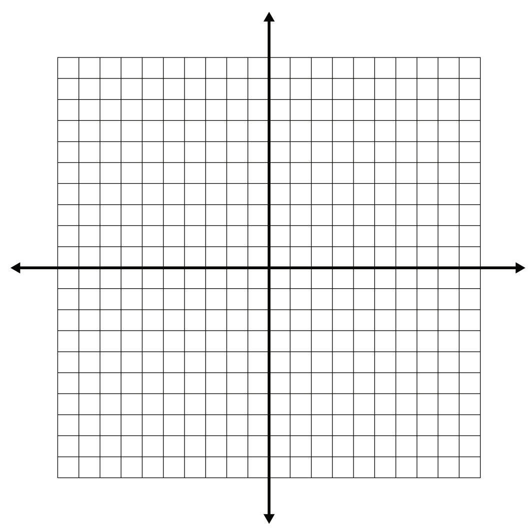
Zeros, roots, and x-intercepts

a) Define "*zero of a polynomial function*". Determine if each value is a zero of $P(x) = x^2 - 4x - 5$.

i) -1 ii) 3

b) Find the zeros of $P(x) = x^2 - 4x - 5$ by solving for the roots of the related equation, $P(x) = 0$.

c) Use a graphing calculator to graph $P(x) = x^2 - 4x - 5$. How are the zeros of the polynomial related to the x-intercepts of the graph?



d) How do you know when to describe solutions as zeros, roots, or x-intercepts?

Example 4

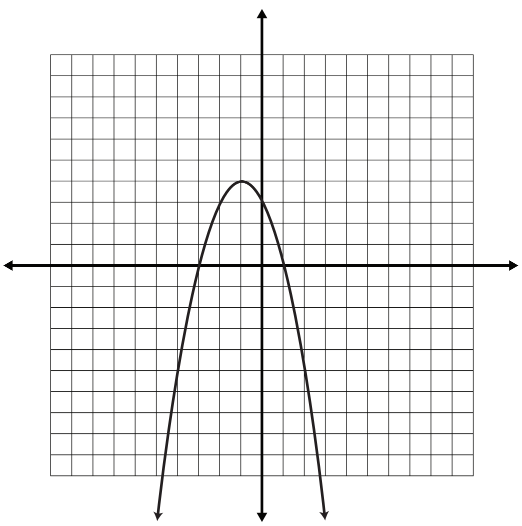
Multiplicity of Zeros in a Polynomial Function.

Multiplicity

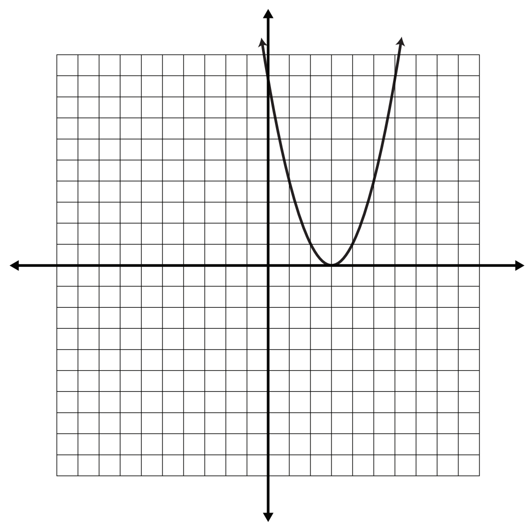
a) Define "*multiplicity of a zero*".

For the graphs in parts (b - e), determine the zeros and state each zero's multiplicity.

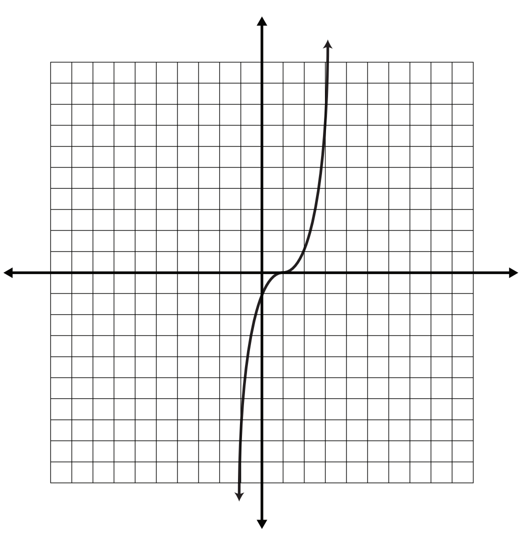
b) $P(x) = -(x + 3)(x - 1)$



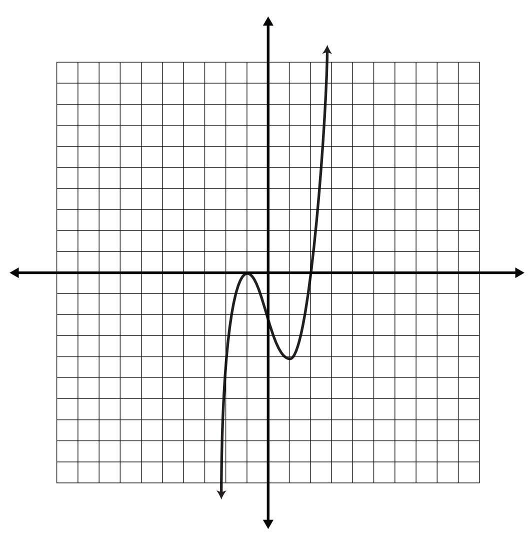
c) $P(x) = (x - 3)^2$

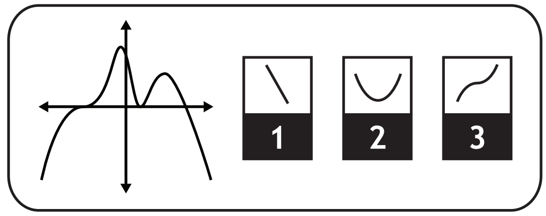


d) $P(x) = (x - 1)^3$



e) $P(x) = (x + 1)^2(x - 2)$



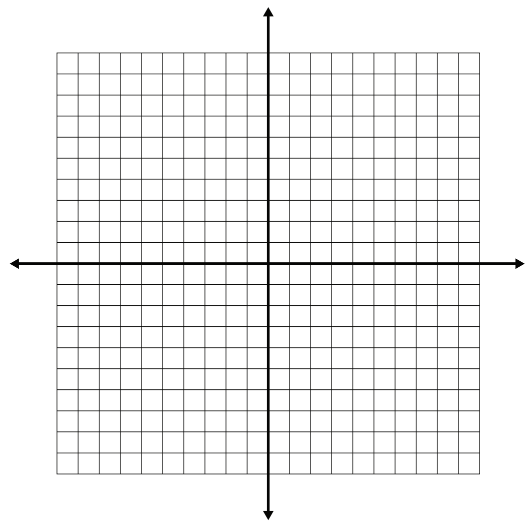


Example 5 Find the requested data for each polynomial function, then use this information to sketch the graph.

Graphing Polynomials

a) $P(x) = \frac{1}{2}(x - 5)(x + 3)$ *Quadratic polynomial with a positive leading coefficient.*

i) Find the zeros and their multiplicities.



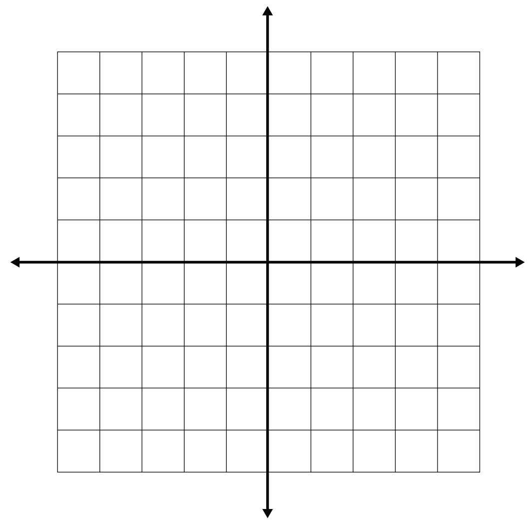
ii) Find the y-intercept.

iii) Describe the end behaviour.

iv) What other points are required to draw the graph accurately?

b) $P(x) = -x^2(x + 1)$ *Cubic polynomial with a negative leading coefficient.*

i) Find the zeros and their multiplicities.



ii) Find the y-intercept.

iii) Describe the end behaviour.

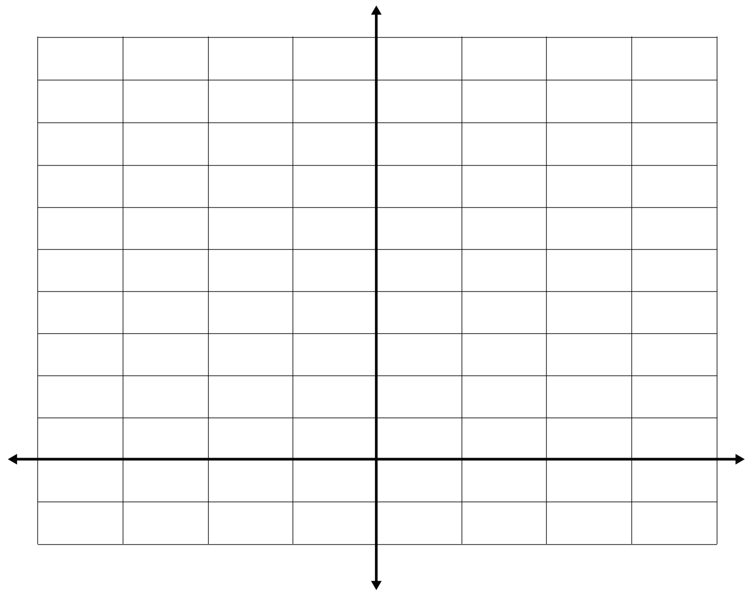
iv) What other points are required to draw the graph accurately?

Example 6

Find the requested data for each polynomial function, then use this information to sketch the graph.

Graphing Polynomials

a) $P(x) = (x - 1)^2(x + 2)^2$ *Quartic polynomial with a positive leading coefficient.*



i) Find the zeros and their multiplicities.

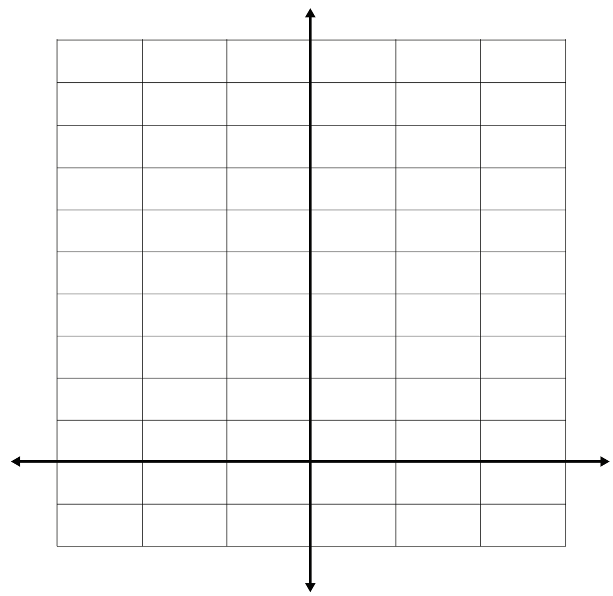
ii) Find the y-intercept.

iii) Describe the end behaviour.

iv) What other points are required to draw the graph accurately?

b) $P(x) = x(x + 1)^3(x - 2)^2$ *Sixth-degree polynomial with a positive leading coefficient.*

i) Find the zeros and their multiplicities.



ii) Find the y-intercept.

iii) Describe the end behaviour.

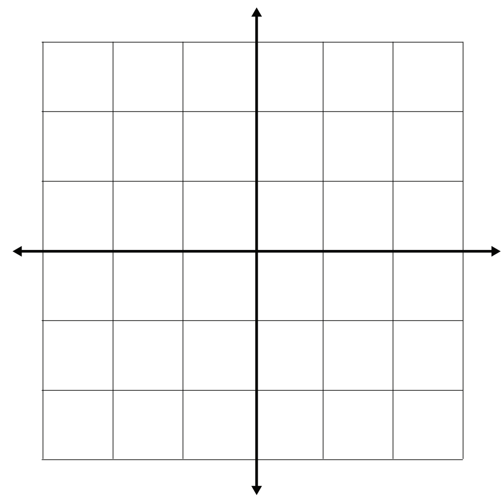
iv) What other points are required to draw the graph accurately?

Example 7

Find the requested data for each polynomial function, then use this information to sketch the graph.

Graphing Polynomials

a) P(x) = -(2x - 1)(2x + 1) *Quadratic polynomial with a negative leading coefficient.*



i) Find the zeros and their multiplicities.

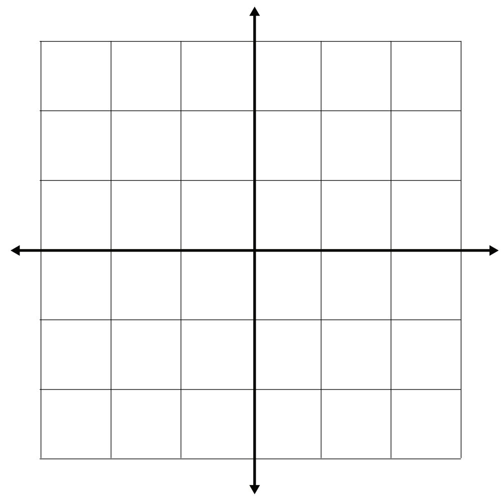
ii) Find the y-intercept.

iii) Describe the end behaviour.

iv) What other points are required to draw the graph accurately?

b) $P(x) = x(4x - 3)(3x + 2)$ *Cubic polynomial with a positive leading coefficient.*

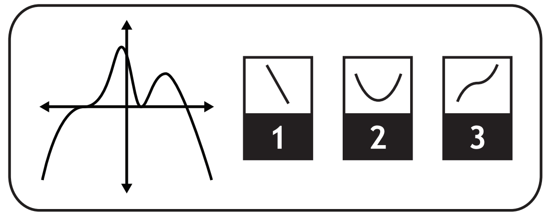
i) Find the zeros and their multiplicities.



ii) Find the y-intercept.

iii) Describe the end behaviour.

iv) What other points are required to draw the graph accurately?

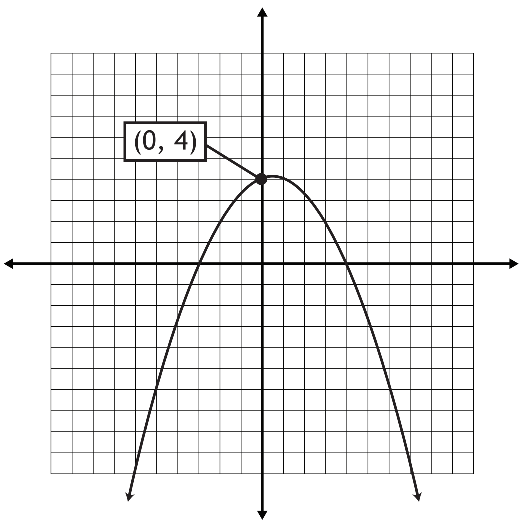


Example 8

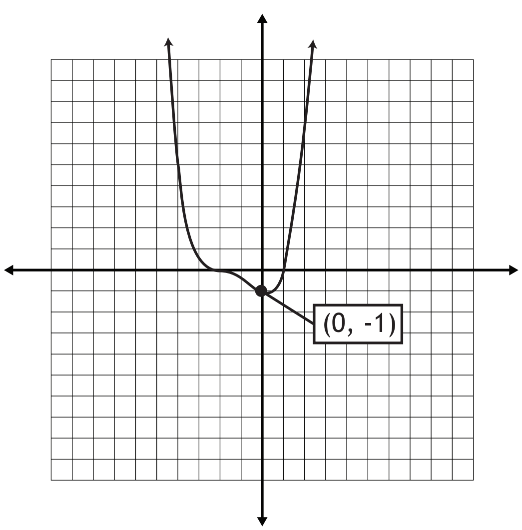
Determine the polynomial function corresponding to each graph. You may leave your answer in factored form.

Finding a Polynomial From its Graph

a)



b)

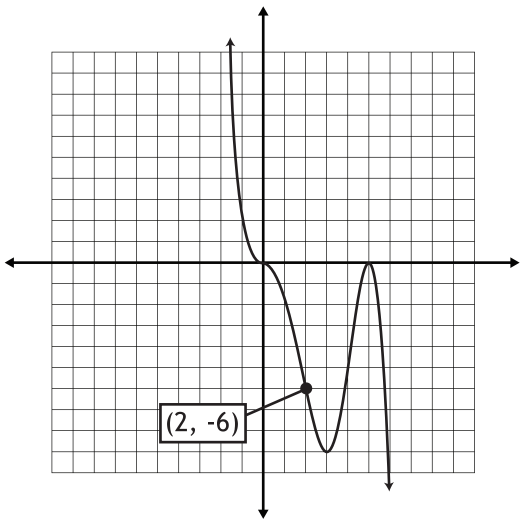


Example 9

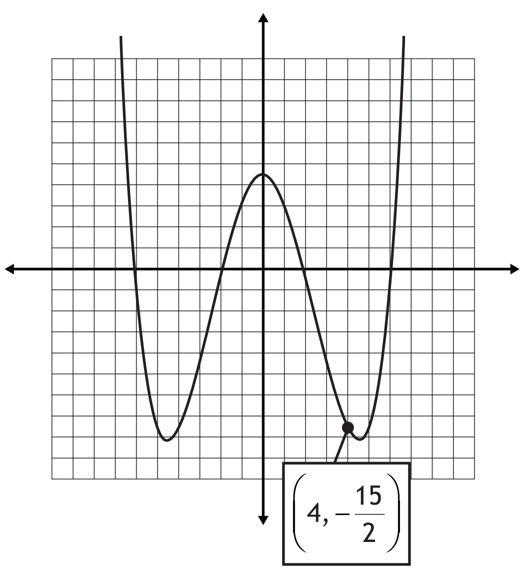
Determine the polynomial function corresponding to each graph. You may leave your answer in factored form.

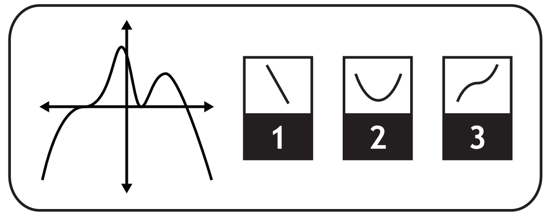
Finding a Polynomial From its Graph

a)



b)

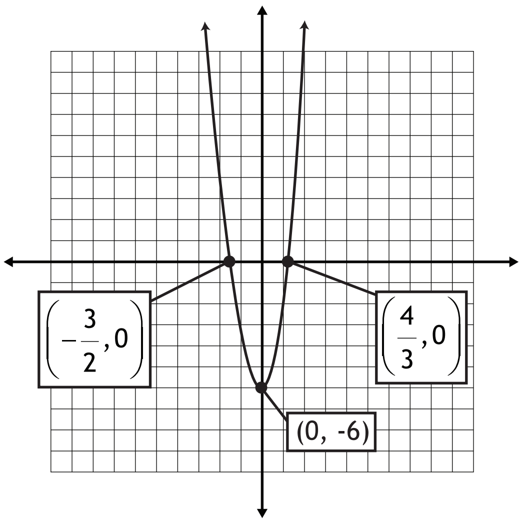




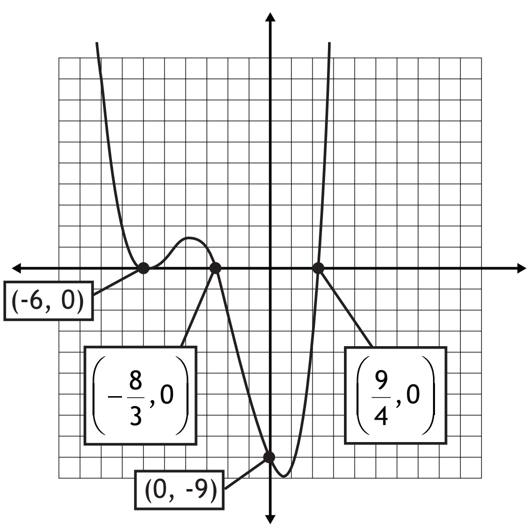
Example 10 Determine the polynomial function corresponding to each graph. You may leave your answer in factored form.

Finding a Polynomial From its Graph

a)



b)



Example 11

Use a graphing calculator to graph each polynomial function. Find window settings that clearly show the important features of each graph *(x-intercepts, y-intercept, and end behaviour)*.

Graphing Polynomials with Technology

a) $P(x) = x^2 - 2x - 168$

Draw the graph.

b) $P(x) = x^3 + 7x^2 - 44x$

Draw the graph.

c) $P(x) = x^3 - 16x^2 - 144x + 1152$

Draw the graph.

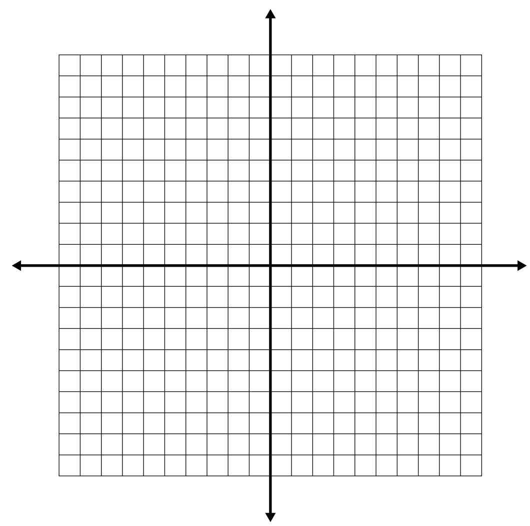
Example 12

Given the characteristics of a polynomial function, draw the graph and derive the actual function.

Graph and Write the Polynomial

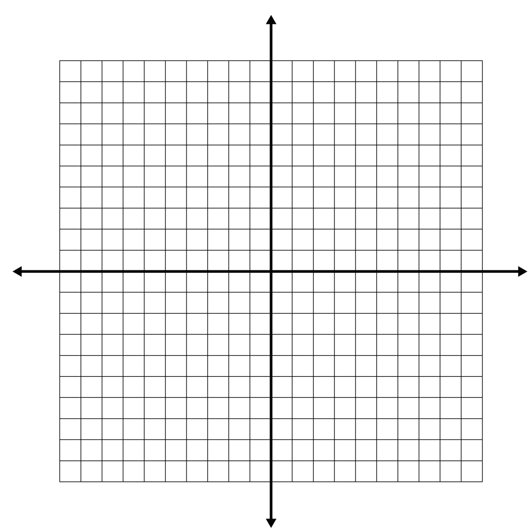
a) Characteristics of P(x):

x-intercepts: (-1, 0) and (3, 0)
sign of leading coefficient: (+)
polynomial degree: 4
relative maximum at (1, 8)



b) Characteristics of P(x):

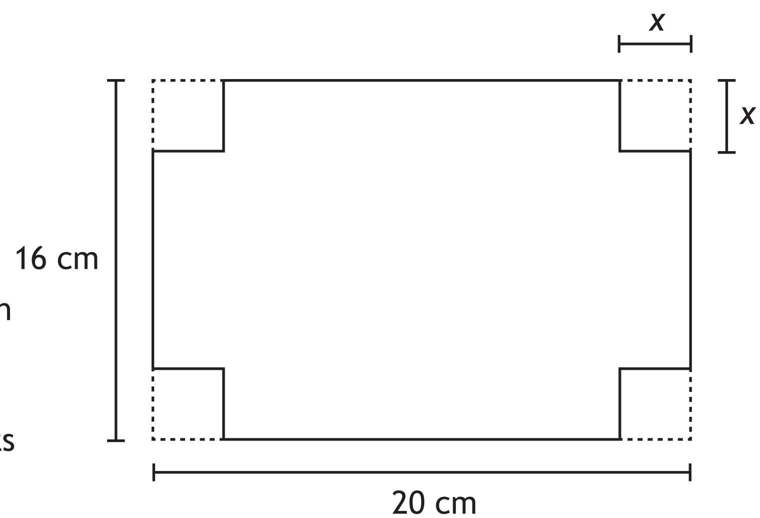
x-intercepts: (-3, 0), (1, 0), and (4, 0)
sign of leading coefficient: (-)
polynomial degree: 3
y-intercept at: $\left(0, -\frac{3}{2}\right)$

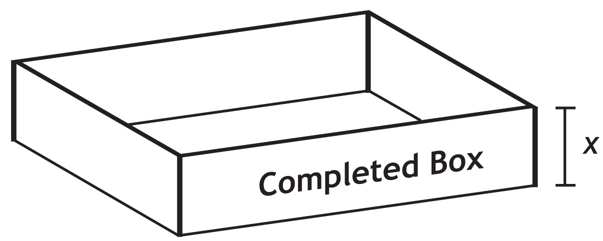


Example 13

A box with no lid can be made by cutting out squares from each corner of a rectangular piece of cardboard and folding up the sides.

A particular piece of cardboard has a length of 20 cm and a width of 16 cm. The side length of a corner square is x.





a) Derive a polynomial function that represents the volume of the box.

b) What is an appropriate domain for the volume function?

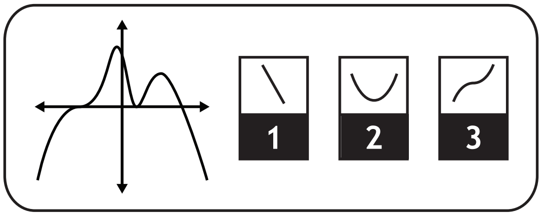
c) Use a graphing calculator to draw the graph of the function. Indicate your window settings.

Draw the graph.

d) What should be the side length of a corner square if the volume of the box is maximized?

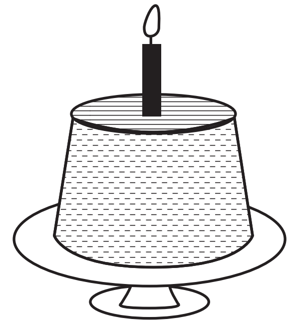
e) For what values of x is the volume of the box greater than 200 cm^3?

Draw the graph.



Example 14

Three students share a birthday on the same day. Quinn and Ralph are the same age, but Audrey is two years older. The product of their ages is 11548 greater than the sum of their ages.



a) Find polynomial functions that represent the age product and age sum.

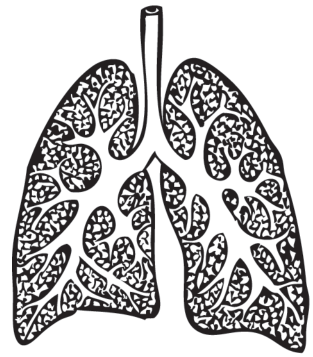
b) Write a polynomial equation that can be used to find the age of each person.

c) Use a graphing calculator to solve the polynomial equation from part (b). Indicate your window settings. How old is each person?

Draw the graph.

Lesson Notes

Example 15



The volume of air flowing into the lungs during a breath can be represented by the polynomial function $V(t) = -0.041t^3 + 0.181t^2 + 0.202t$, where V is the volume in litres and t is the time in seconds.

a) Use a graphing calculator to graph V(t). State your window settings.

Draw the graph.

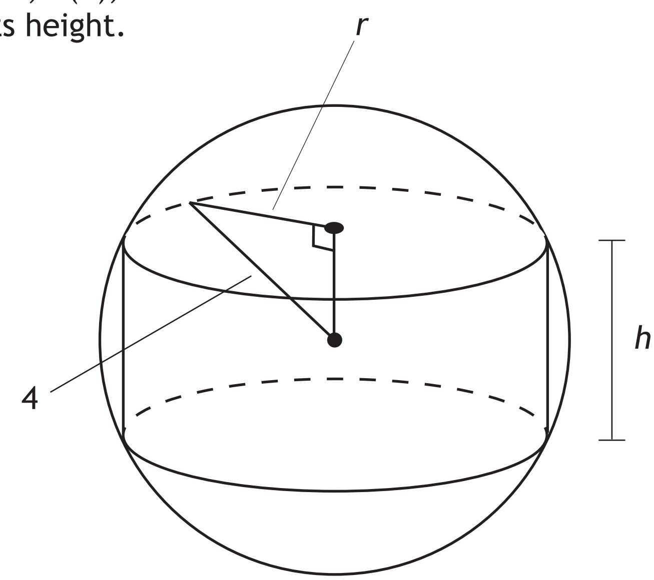
b) What is the maximum volume of air inhaled into the lung? At what time during the breath does this occur?

c) How many seconds does it take for one complete breath?

d) What percentage of the breath is spent inhaling?

Example 16

A cylinder with a radius of *r* and a height of *h* is inscribed within a sphere that has a radius of 4 units. Derive a polynomial function, V(h), that expresses the volume of the cylinder as a function of its height.



$V_{cylinder} = \pi r^2 h$

1	3	-4	-5	2
–	↓	3	-7	2
	3	-7	2	0

Polynomial, Radical, and Rational Functions
LESSON TWO - *Polynomial Division*
Lesson Notes

Example 1 Divide $(x^3 + 2x^2 - 5x - 6)$ by $(x + 2)$ using long division and answer the related questions.

Long & Synthetic Polynomial Division

a) $x + 2 \overline{\big)\, x^3 + 2x^2 - 5x - 6}$

b) Label the division components *(dividend, divisor, quotient, remainder)* in your work for part (a).

c) Express the division using the division theorem, $P(x) = Q(x) \bullet D(x) + R$. Verify the division theorem by checking that the left side and right side are equivalent.

d) Another way to represent the division theorem is $\frac{P(x)}{D(x)} = Q(x) + \frac{R}{D(x)}$.

Express the division using this format.

e) Synthetic division is a quicker way of dividing than long division. Divide $(x^3 + 2x^2 - 5x - 6)$ by $(x + 2)$ using synthetic division and express the result in the form $\frac{P(x)}{D(x)} = Q(x) + \frac{R}{D(x)}$.

1	3	-4	-5	2
-	↓	3	-7	2
	3	-7	2	0

Example 2

Divide using long division.
Express answers in the form $\frac{P(x)}{D(x)} = Q(x) + \frac{R}{D(x)}$.

Polynomial Division
(Long Division)

a) $(3x^3 - 4x^2 + 2x - 1) \div (x + 1)$

b) $\frac{x^3 - 3x - 2}{x - 2}$

c) $(x^3 - 1) \div (x + 2)$

1	|	3	-4	-5	2
-	|	↓	3	-7	2
		3	-7	2	0

Polynomial Division
(Synthetic Division)

Example 3

Divide using synthetic division.

Express answers in the form $\frac{P(x)}{D(x)} = Q(x) + \frac{R}{D(x)}$.

a) $(3x^3 - x - 3) \div (x - 1)$

b) $\frac{3x^4 + 5x^3 + 3x - 2}{x + 2}$

c) $(2x^4 - 7x^2 + 4) \div (x - 1)$

Example 4

Polynomial division only requires long or synthetic division when factoring is not an option. Try to divide each of the following polynomials by factoring first, using long or synthetic division as a backup.

Polynomial Division
(Factoring)

a) $\dfrac{x^2 - 5x + 6}{x - 3}$

b) $(6x - 4) \div (3x - 2)$

c) $(x^4 - 16) \div (x^2 + 4)$

d) $\dfrac{x^3 + 2x^2 - 3x}{x - 3}$

Example 5

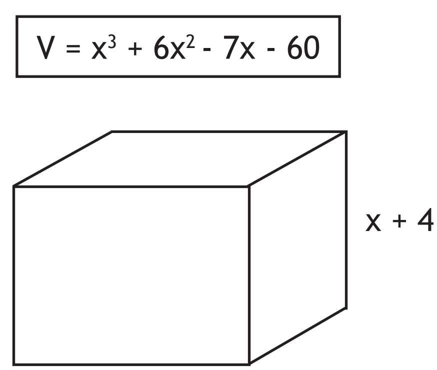
When $3x^3 - 4x^2 + ax + 2$ is divided by $x + 1$, the quotient is $3x^2 - 7x + 2$ and the remainder is zero. Solve for a using two different methods.

a) Solve for a using synthetic division.

b) Solve for a using $P(x) = Q(x) \bullet D(x) + R$.

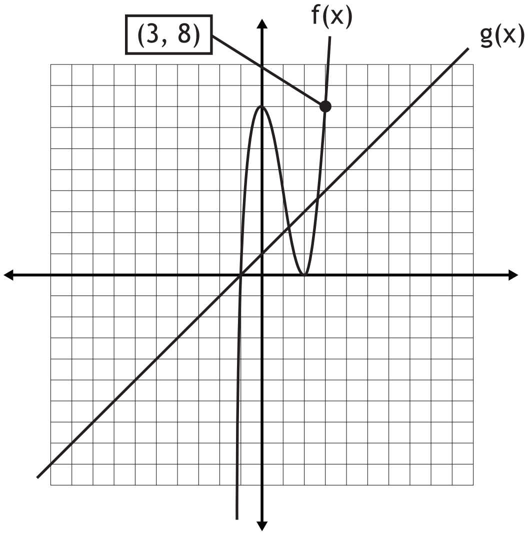
Example 6

A rectangular prism has a volume of $x^3 + 6x^2 - 7x - 60$. If the height of the prism is $x + 4$, determine the dimensions of the base.



Example 7

The graphs of f(x) and g(x) are shown below.



a) Determine the polynomial corresponding to f(x).

b) Determine the equation of the line corresponding to g(x).

Recall that the equation of a line can be found using y = mx + b, where m is the slope of the line and the y-intercept is (0, b).

c) Determine Q(x) = f(x) ÷ g(x) and draw the graph of Q(x).

1	3	-4	-5	2
-	↓	3	-7	2
	3	-7	2	0

Example 8 If $f(x) \div g(x) = 4x^2 + 4x - 3 - \frac{6}{x - 1}$, determine f(x) and g(x).

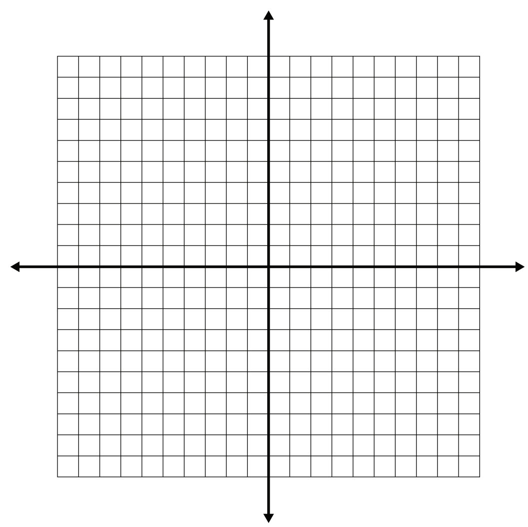
Example 9

The Remainder Theorem

The Remainder Theorem

a) Divide $2x^3 - x^2 - 3x - 2$ by $x - 1$ using synthetic division and state the remainder.

b) Draw the graph of $P(x) = 2x^3 - x^2 - 3x - 2$ using technology.
What is the value of $P(1)$?



c) How does the remainder in part (a) compare with the value of $P(1)$ in part (b)?

d) Using the graph from part (b), find the remainder when $P(x)$ is divided by:

i) $x - 2$ ii) x iii) $x + 1$

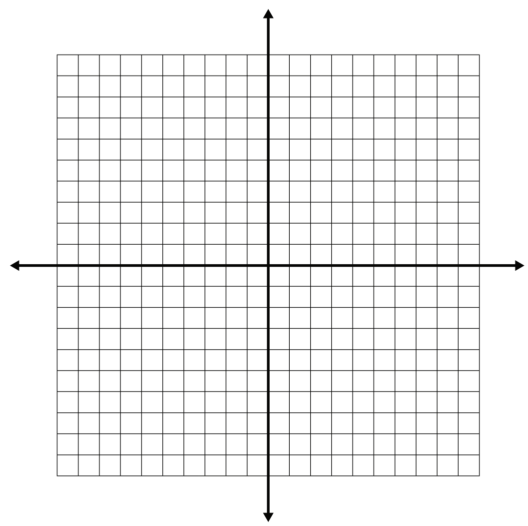
e) Define the remainder theorem.

Example 10 The Factor Theorem

The Factor Theorem

a) Divide $x^3 - 3x^2 + 4x - 2$ by $x - 1$ using synthetic division and state the remainder.

b) Draw the graph of $P(x) = x^3 - 3x^2 + 4x - 2$ using technology.
What is the remainder when $P(x)$ is divided by $x - 1$?



c) How does the remainder in part (a) compare with the value of $P(1)$ in part (b)?

d) Define the factor theorem.

e) Draw a diagram that illustrates the relationship between the remainder theorem and the factor theorem.

Example 11

For each division, use the remainder theorem to find the remainder. Use the factor theorem to determine if the divisor is a factor of the polynomial.

Is the Divisor a Factor?

a) $(x^3 - 1) \div (x + 1)$

b) $\frac{x^4 - 2x^2 + 3x - 4}{x + 2}$

c) $(3x^3 + 8x^2 - 1) \div (3x - 1)$

d) $\frac{2x^4 + 3x^3 - 4x - 9}{2x + 3}$

1	3	-4	-5	2
-	↓	3	-7	2
	3	-7	2	0

Example 12

Use the remainder theorem to find the value of *k* in each polynomial.

One-Unknown Problems

a) $(kx^3 - x - 3) \div (x - 1)$ *Remainder = -1*

b) $\dfrac{3x^3 - 6x^2 + 2x + k}{x - 2}$ *Remainder = -3*

c) $(2x^3 + 3x^2 + kx - 3) \div (2x + 5)$ *Remainder = 2*

d) $\dfrac{2x^3 + kx^2 - x + 6}{2x - 3}$ *(2x - 3 is a factor)*

Example 13

Two-Unknown Problems

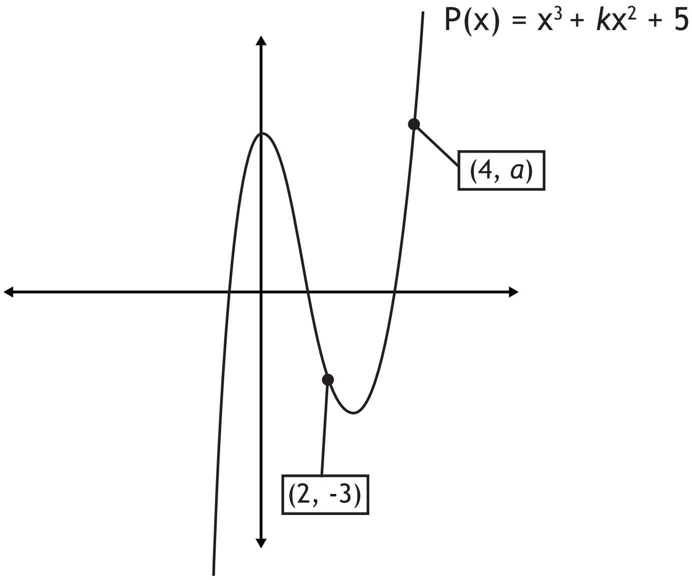
When $3x^3 + mx^2 + nx + 2$ is divided by $x + 2$, the remainder is 8. When the same polynomial is divided by $x - 1$, the remainder is 2. Determine the values of m and n.

Example 14

When $2x^3 + mx^2 + nx - 6$ is divided by $x - 2$, the remainder is 20. The same polynomial has a factor of $x + 2$. Determine the values of m and n.

Example 15

Given the graph of $P(x) = x^3 + kx^2 + 5$ and the point (2, -3), determine the value of a on the graph.



Example 1

The Integral Zero Theorem

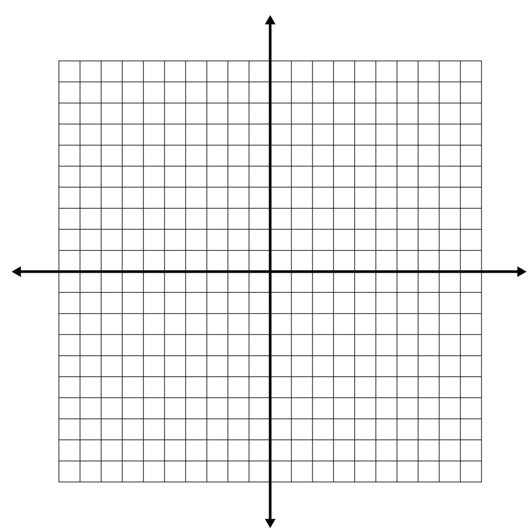
Integral Zero Theorem

a) Define the *integral zero theorem*. How is this theorem useful in factoring a polynomial?

b) Using the integral zero theorem, find potential zeros of the polynomial $P(x) = x^3 + x^2 - 5x + 3$.

c) Which potential zeros from part (b) are actually zeros of the polynomial?

d) Use technology to draw the graph of $P(x) = x^3 + x^2 - 5x + 3$. How do the x-intercepts of the graph compare to the zeros of the polynomial function?



e) Use the graph from part (d) to factor $P(x) = x^3 + x^2 - 5x + 3$.

Example 2

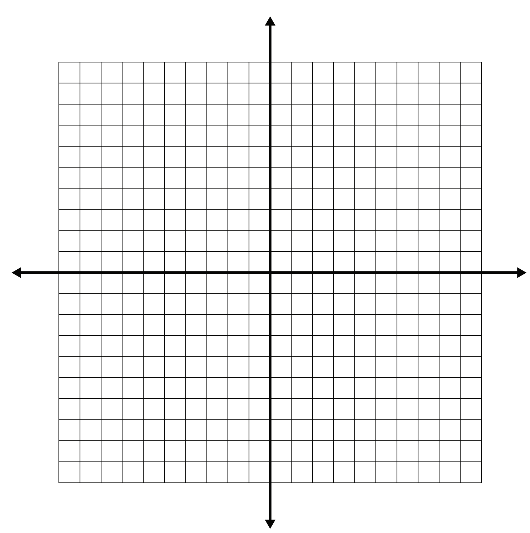
Factor and graph $P(x) = x^3 + 3x^2 - x - 3$.

Polynomial Factoring

a) Factor algebraically using the integral zero theorem.

b) Use technology to graph the polynomial. Can the polynomial be factored using just the graph?

c) Can $P(x)$ be factored any other way?



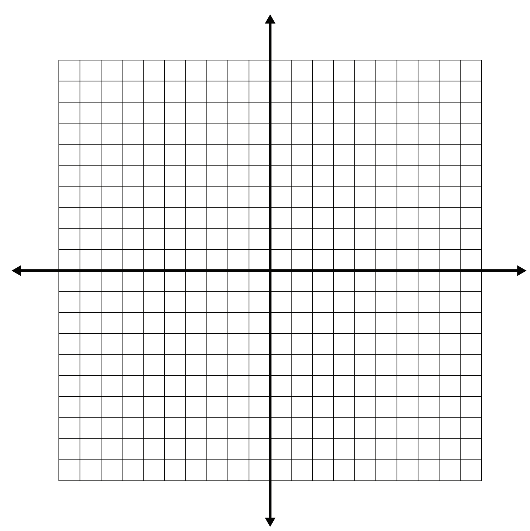
Example 3 Factor and graph $P(x) = 2x^3 - 6x^2 + x - 3$

Polynomial Factoring

a) Factor algebraically using the integral zero theorem.

b) Use technology to graph the polynomial. Can the polynomial be factored using just the graph?

c) Can P(x) be factored any other way?



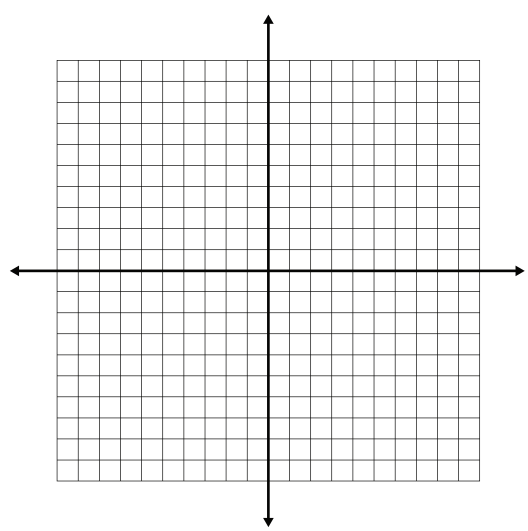
Example 4 Factor and graph $P(x) = x^3 - 3x + 2$

Polynomial Factoring

a) Factor algebraically using the integral zero theorem.

b) Use technology to graph the polynomial. Can the polynomial be factored using just the graph?

c) Can P(x) be factored any other way?



Example 5

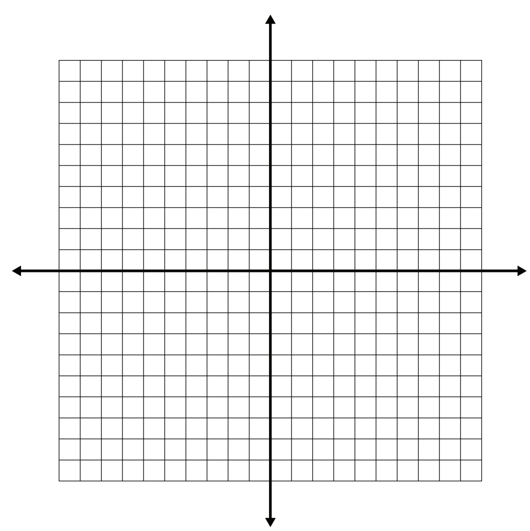
Factor and graph $P(x) = x^3 - 8$

Polynomial Factoring

a) Factor algebraically using the integral zero theorem.

b) Use technology to graph the polynomial. Can the polynomial be factored using just the graph?

c) Can $P(x)$ be factored any other way?



Lesson Notes

$x^3 - 5x^2 + 2x + 8$ ⟷ $(x + 1)(x - 2)(x - 4)$

Example 6

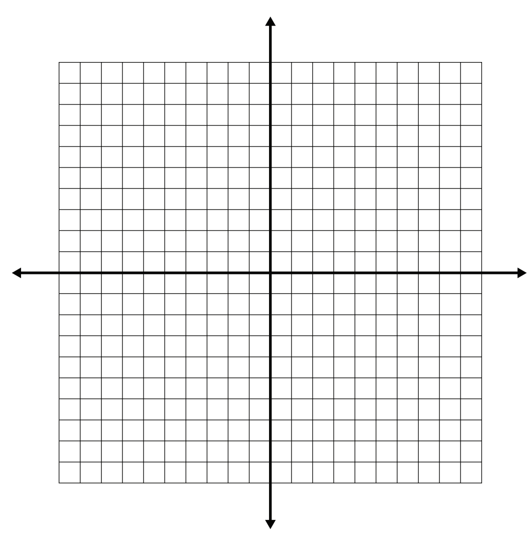
Factor and graph $P(x) = x^3 - 2x^2 - x - 6$

Polynomial Factoring

a) Factor algebraically using the integral zero theorem.

b) Use technology to graph the polynomial. Can the polynomial be factored using just the graph?

c) Can P(x) be factored any other way?



Example 7

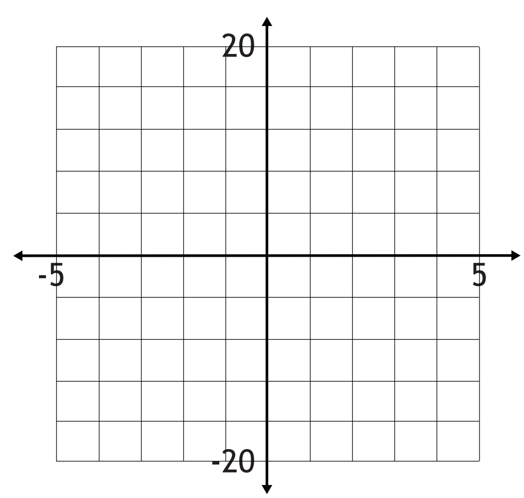
Factor and graph $P(x) = x^4 - 16$

Polynomial Factoring

a) Factor algebraically using the integral zero theorem.

b) Use technology to graph the polynomial. Can the polynomial be factored using just the graph?

c) Can P(x) be factored any other way?



Polynomial, Radical, and Rational Functions
LESSON THREE - *Polynomial Factoring*
Lesson Notes

$x^3 - 5x^2 + 2x + 8$

$(x + 1)(x - 2)(x - 4)$

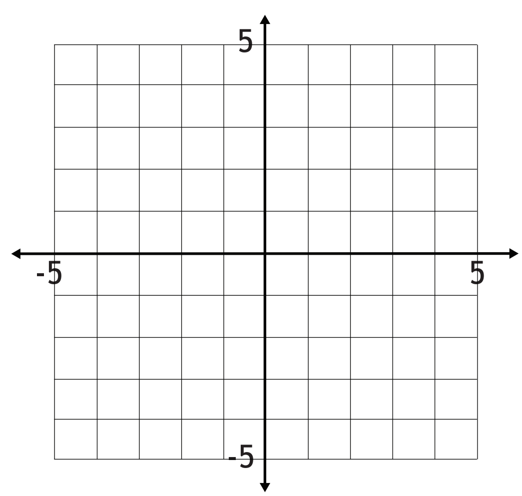
Example 8 Factor and graph $P(x) = x^5 - 3x^4 - 5x^3 + 27x^2 - 32x + 12$

Polynomial Factoring

a) Factor algebraically using the integral zero theorem.

b) Use technology to graph the polynomial. Can the polynomial be factored using just the graph?

c) Can P(x) be factored any other way?

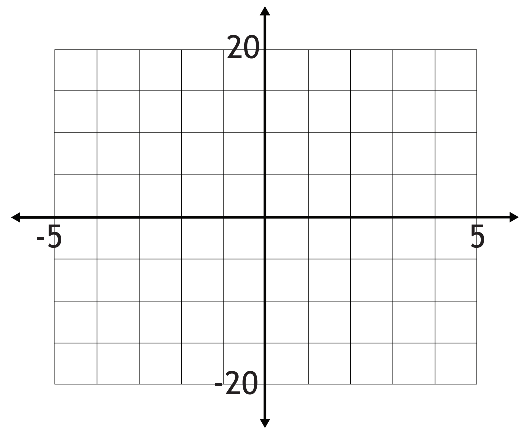


Example 9

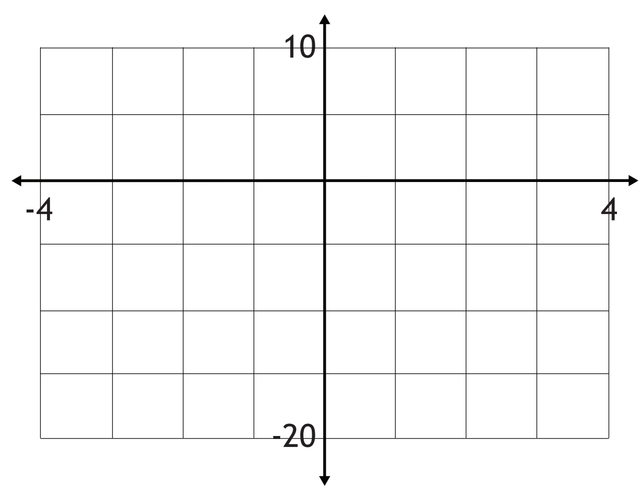
Given the zeros of a polynomial and a point on its graph, find the polynomial function. You may leave the polynomial in factored form. Sketch each graph.

Find the Polynomial Function

a) P(x) has zeros of -4, 0, 0, and 1.
The graph passes through the point (-1, -3).



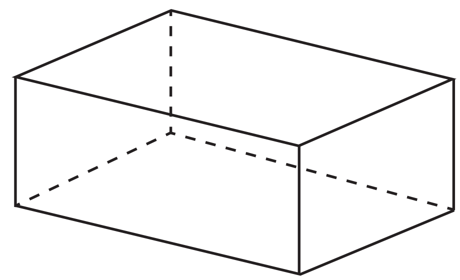
b) P(x) has zeros of -1, -1, and 2.
The graph passes through the point (1, -8).



Example 10

Problem Solving

A rectangular prism has a volume of 1050 cm^3. If the height of the prism is 3 cm less than the width of the base, and the length of the base is 5 cm greater than the width of the base, find the dimensions of the rectangular prism. Solve algebraically.



Example 11

Find three consecutive integers with a product of -336. Solve algebraically.

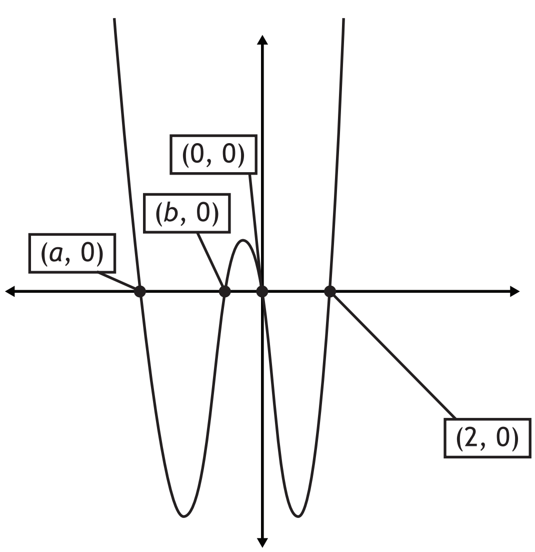
Example 12

Problem Solving

If k, $3k$, and $-3k/2$ are zeros of $P(x) = x^3 - 5x^2 - 6kx + 36$, and $k > 0$, find k and write the factored form of the polynomial.

Example 13

Given the graph of $P(x) = x^4 + 2x^3 - 5x^2 - 6x$ and various points on the graph, determine the values of a and b. Solve algebraically.



$x^3 - 5x^2 + 2x + 8$

$(x + 1)(x - 2)(x - 4)$

Example 14

Solve each equation algebraically.
Check with a graphing calculator.

Polynomial Equations

a) $x^3 - 3x^2 - 10x + 24 = 0$

b) $3x^3 + 8x^2 + 4x - 1 = 0$

Quadratic Formula

From Math 20-1:
The roots of a quadratic equation with the form $ax^2 + bx + c = 0$ can be found with the ***quadratic formula***:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Lesson Notes

Radical Functions

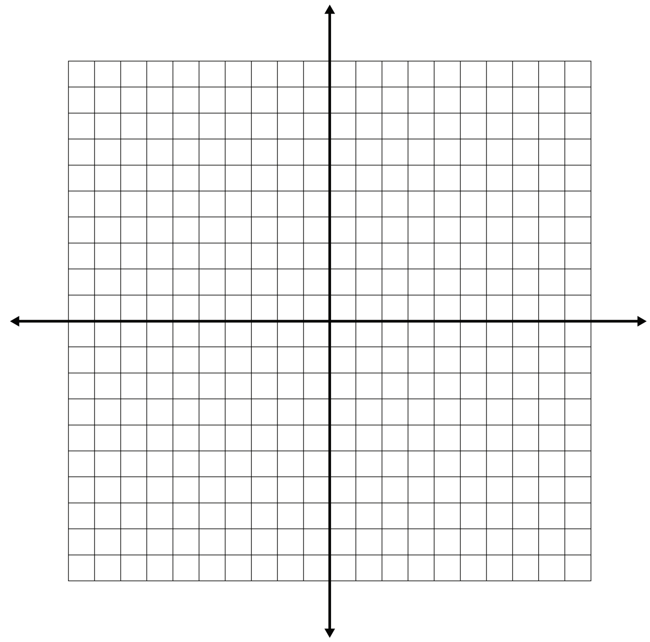
Example 1

Introduction to Radical Functions

a) Fill in the table of values for the function $f(x) = \sqrt{x}$

x	*f(x)*
-1	
0	
1	
4	
9	

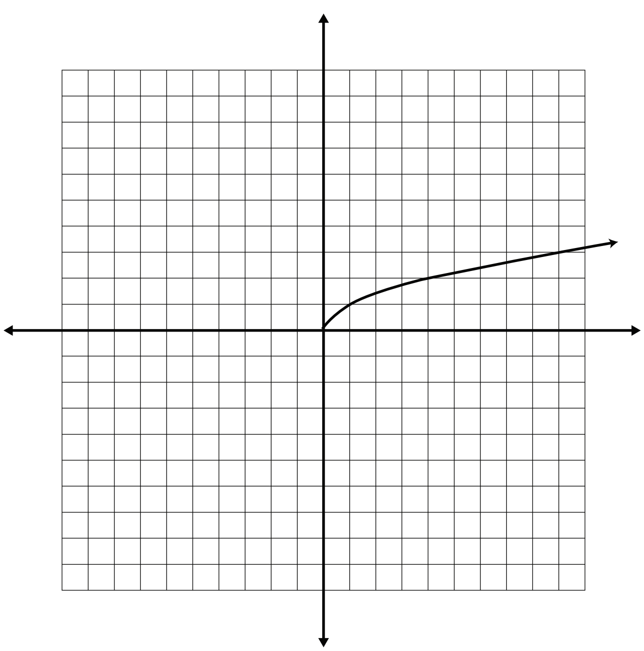
b) Draw the graph of the function $f(x) = \sqrt{x}$ and state the domain and range.



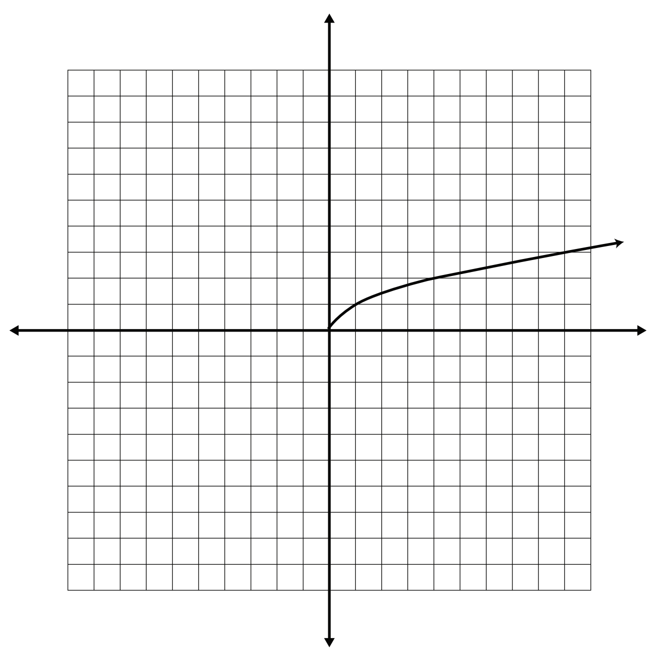
Example 2

Graph each function. The graph of $y = \sqrt{x}$ is provided as a reference.

a) $f(x) = -\sqrt{x}$ ***reflection about the x-axis***



b) $f(x) = \sqrt{-x}$ ***reflection about the y-axis***



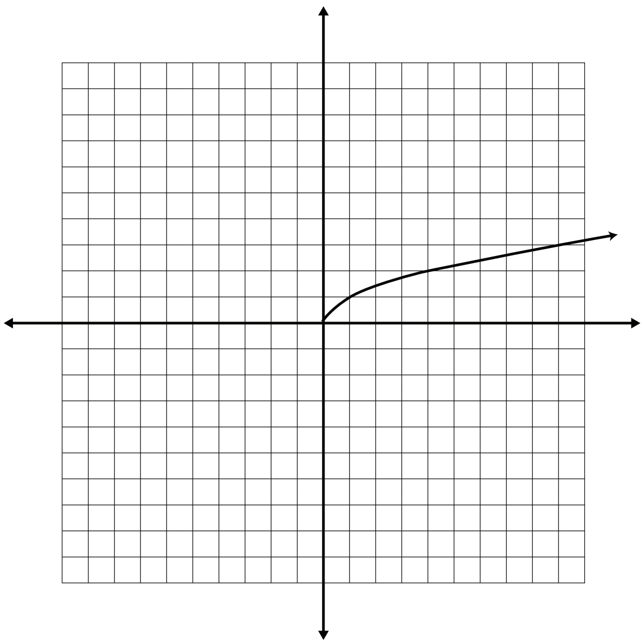
$y = \sqrt{x}$

Example 3

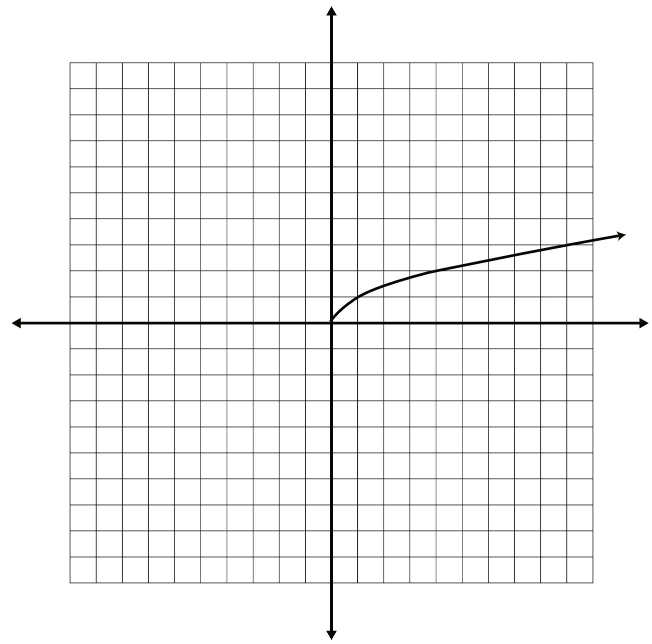
Graph each function.
The graph of $y = \sqrt{x}$ is provided as a reference.

Transformations of Radical Functions

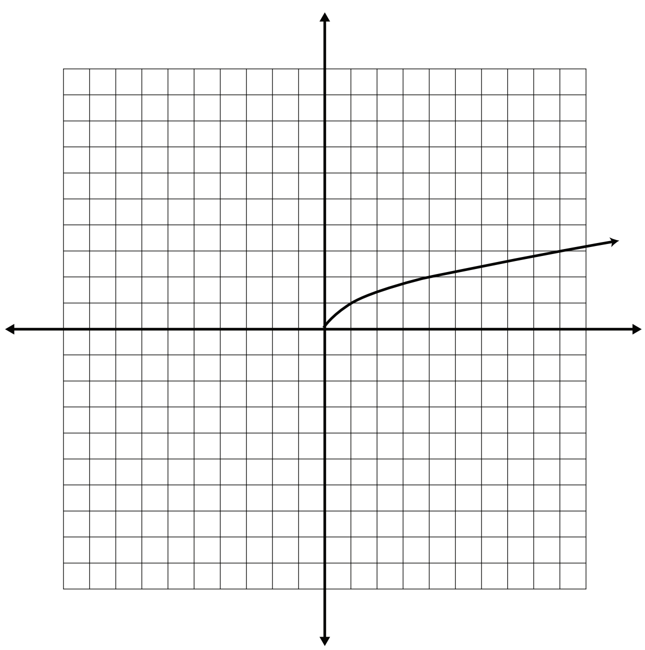
a) $f(x) = 2\sqrt{x}$ *vertical stretch (double)*



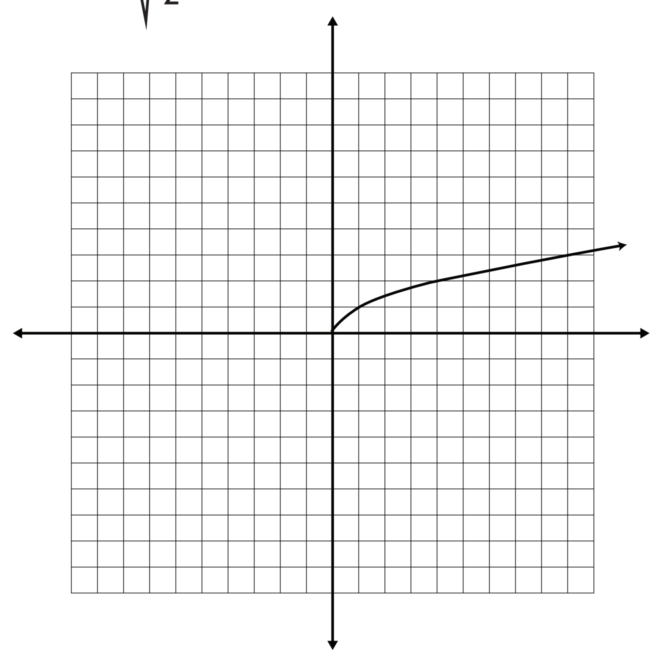
b) $f(x) = \frac{1}{2}\sqrt{x}$ *vertical stretch (half)*

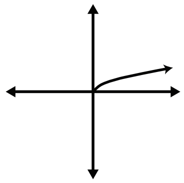


c) $f(x) = \sqrt{2x}$ *horizontal stretch (half)*



d) $f(x) = \sqrt{\frac{1}{2}x}$ *horizontal stretch (double)*





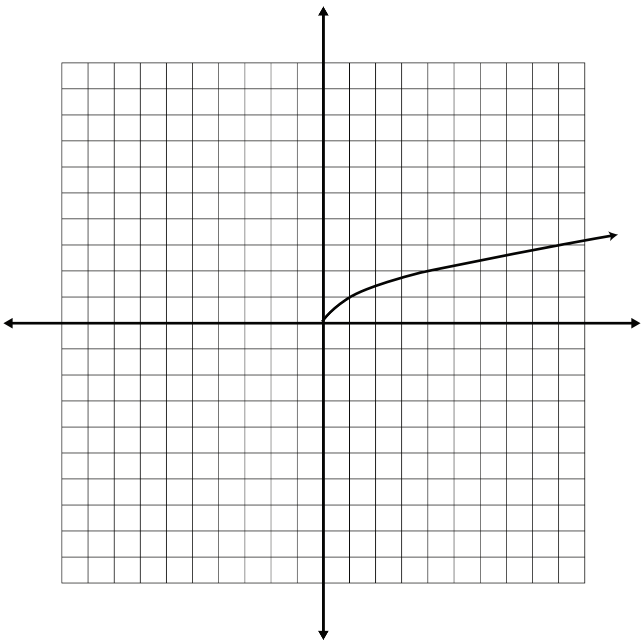
$y = \sqrt{x}$

Example 4

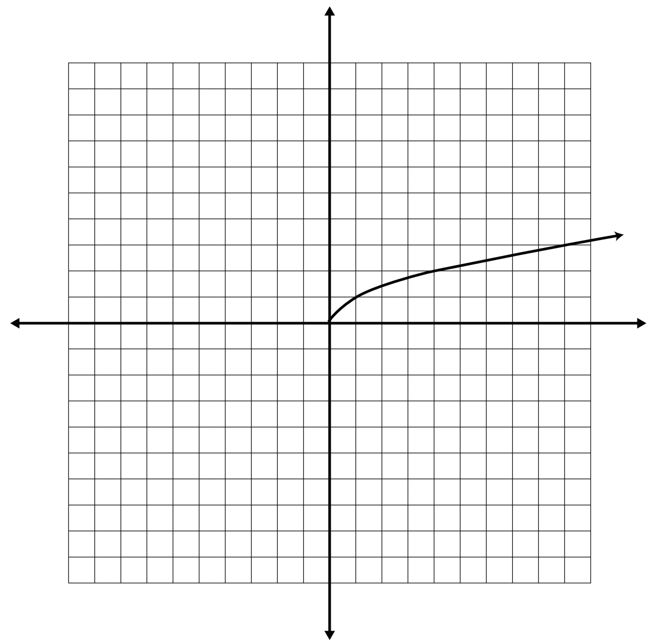
Graph each function.
The graph of $y = \sqrt{x}$ is provided as a reference.

Transformations of Radical Functions

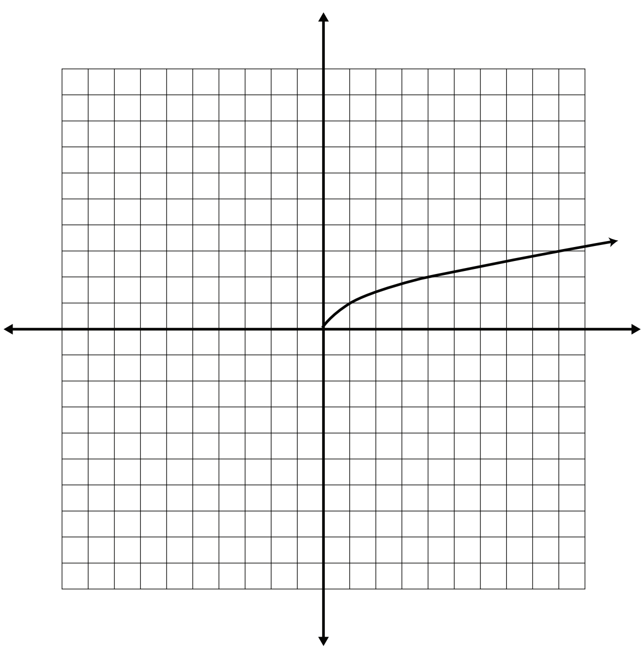
a) $f(x) = \sqrt{x} - 5$ ***vertical translation (down)***



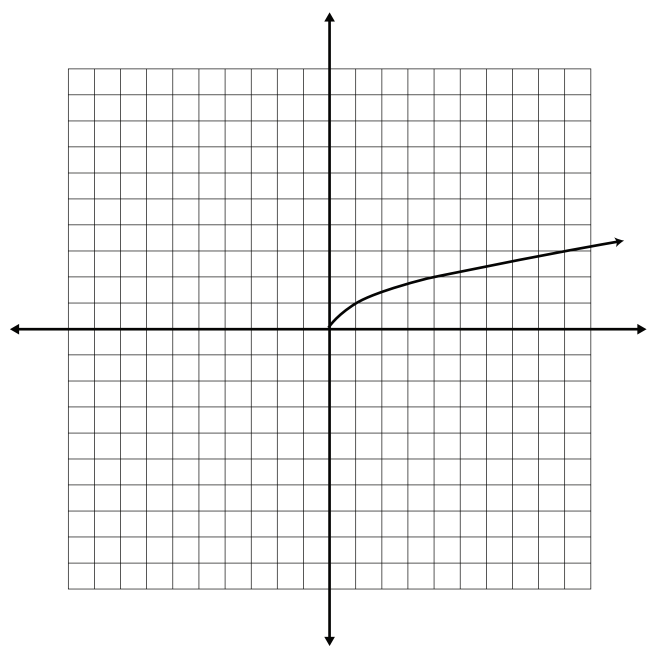
b) $f(x) = \sqrt{x} + 2$ ***vertical translation (up)***

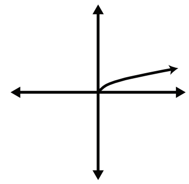


c) $f(x) = \sqrt{x - 1}$ ***horizontal translation (right)***



d) $f(x) = \sqrt{x + 7}$ ***horizontal translation (left)***



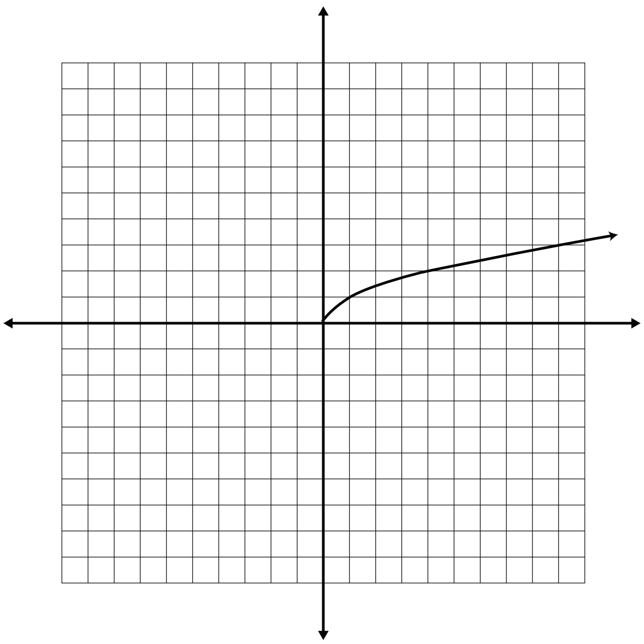


Example 5

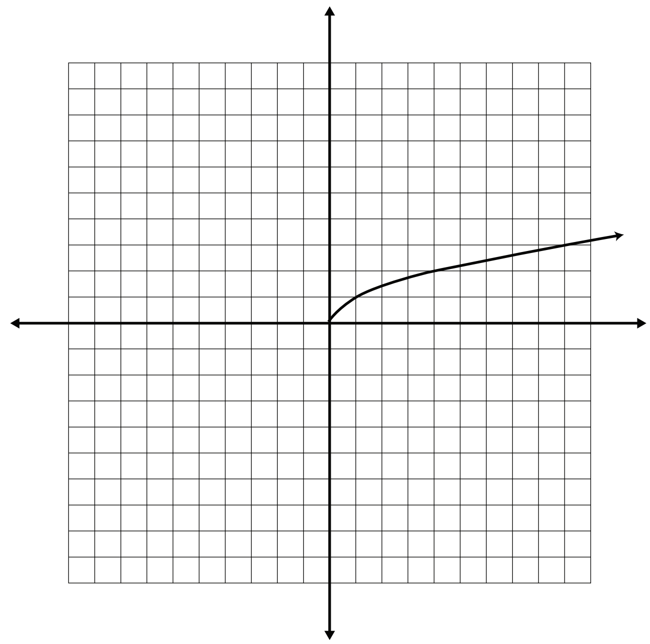
Graph each function.
The graph of $y = \sqrt{x}$ is provided as a reference.

Transformations of Radical Functions

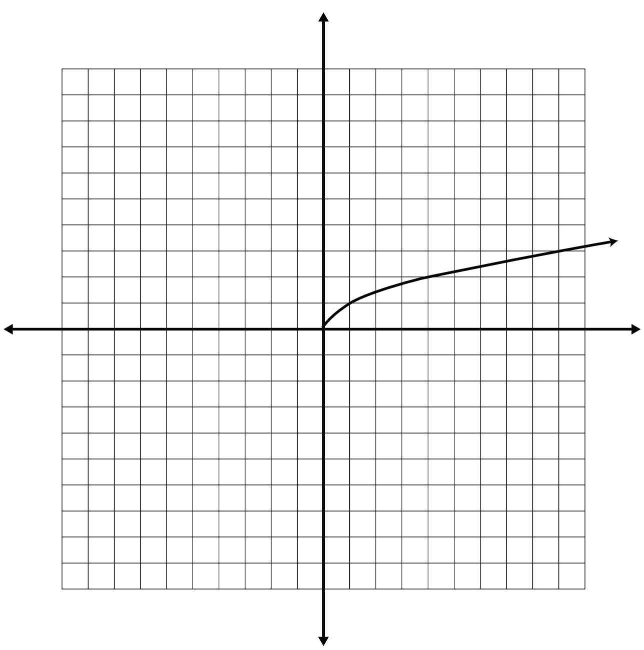
a) $f(x) = \sqrt{x - 3} + 2$



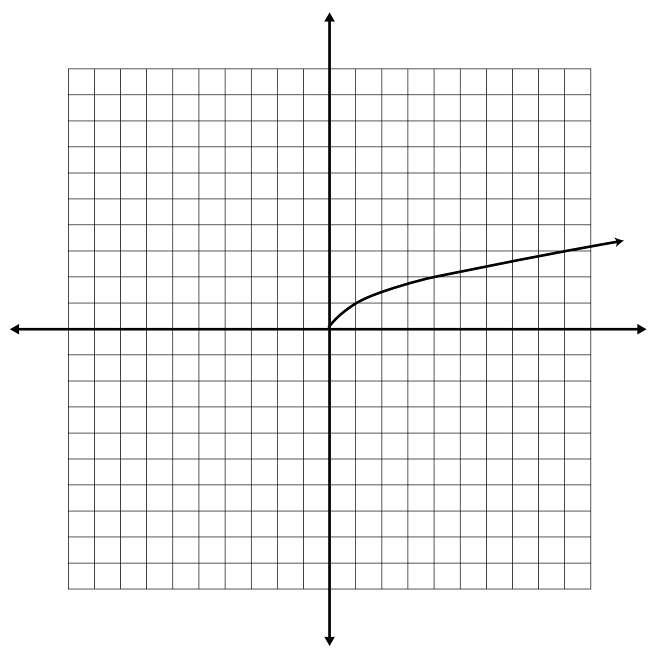
b) $f(x) = 2\sqrt{x + 4}$

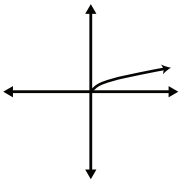


c) $f(x) = -\sqrt{x} - 3$



d) $f(x) = \sqrt{-2x - 4}$



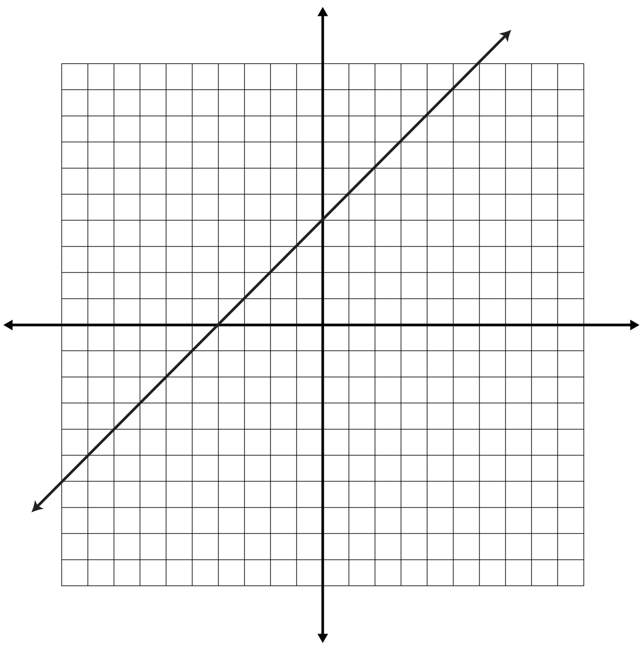
 $y = \sqrt{x}$

Polynomial, Radical, and Rational Functions
LESSON FOUR - *Radical Functions*
Lesson Notes

Example 6 Given the graph of $y = f(x)$, graph $y = \sqrt{f(x)}$ on the same grid.

Square Root of an Existing Function

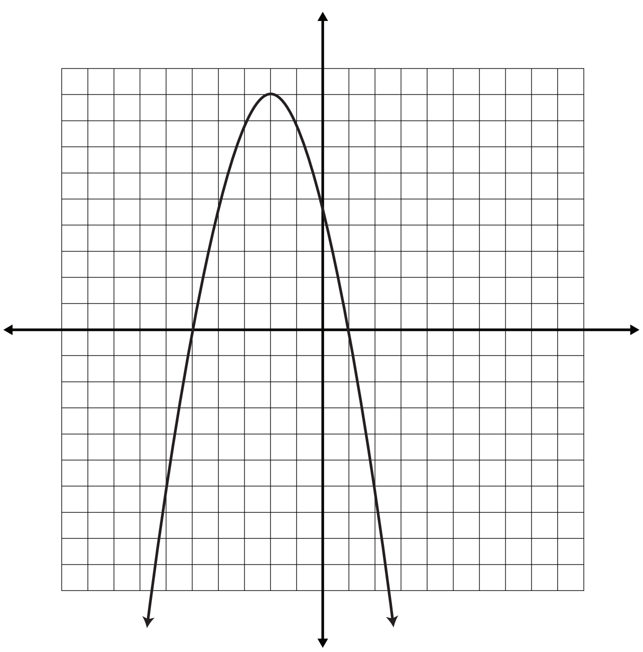
a) $y = x + 4$



Domain & Range for $y = f(x)$

Domain & Range for $y = \sqrt{f(x)}$

b) $y = -(x + 2)^2 + 9$



Domain & Range for $y = f(x)$

Domain & Range for $y = \sqrt{f(x)}$

Set-Builder Notation

A ***set*** is simply a collection of numbers, such as {1, 4, 5}. We use ***set-builder notation*** to outline the rules governing members of a set.

{x | x ε R, x ≥ -1}

State the variable. List conditions on the variable.

In words: "The variable is x, such that x can be any real number with the condition that x ≥ -1".

As a shortcut, set-builder notation can be reduced to just the most important condition.

x ≥ -1

While this resource uses the shortcut for brevity, as set-builder notation is covered in previous courses, Math 30-1 students *are* expected to know how to read and write full set-builder notation.

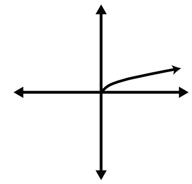
Interval Notation

Math 30-1 students are expected to know that domain and range can be expressed using ***interval notation***.

() - Round Brackets: Exclude point from interval.
[] - Square Brackets: Include point in interval.

Infinity ∞ always gets a round bracket.

Examples: x ≥ -5 becomes [-5, ∞);
1 < x ≤ 4 becomes (1, 4];
x ε R becomes (-∞ , ∞);
-8 ≤ x < 2 or 5 ≤ x < 11 becomes [-8, 2) U [5, 11), where U means "or", or ***union of sets***;
x ε R, x ≠ 2 becomes (-∞ , 2) U (2, ∞);
-1 ≤ x ≤ 3, x ≠ 0 becomes [-1 , 0) U (0, 3].



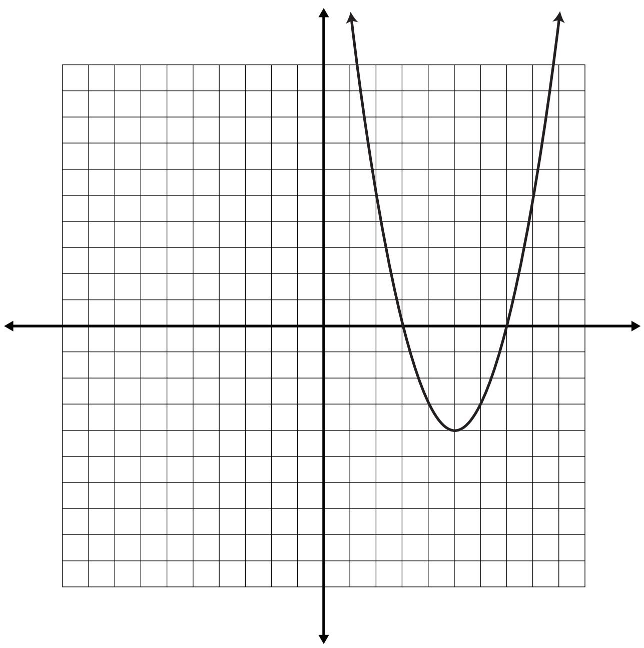
$y = \sqrt{x}$

Example 7

Given the graph of $y = f(x)$, graph $y = \sqrt{f(x)}$ on the same grid.

Square Root of an Existing Function

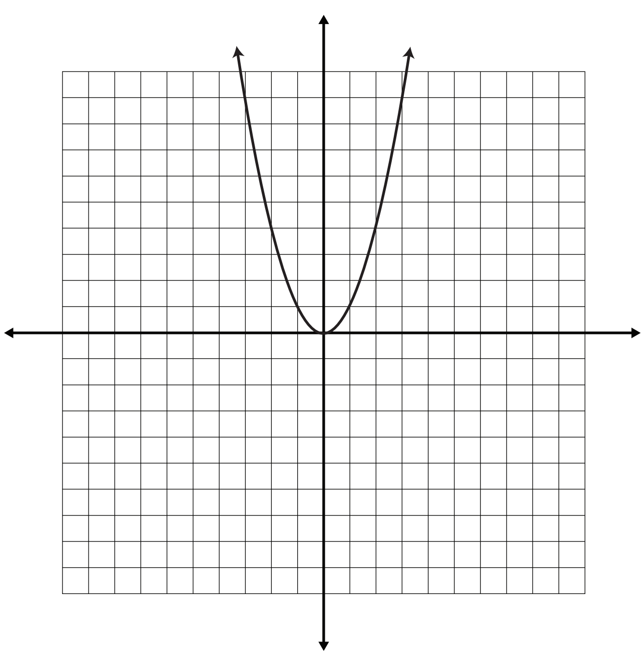
a) $y = (x - 5)^2 - 4$



Domain & Range for $y = f(x)$

Domain & Range for $y = \sqrt{f(x)}$

b) $y = x^2$



Domain & Range for $y = f(x)$

Domain & Range for $y = \sqrt{f(x)}$

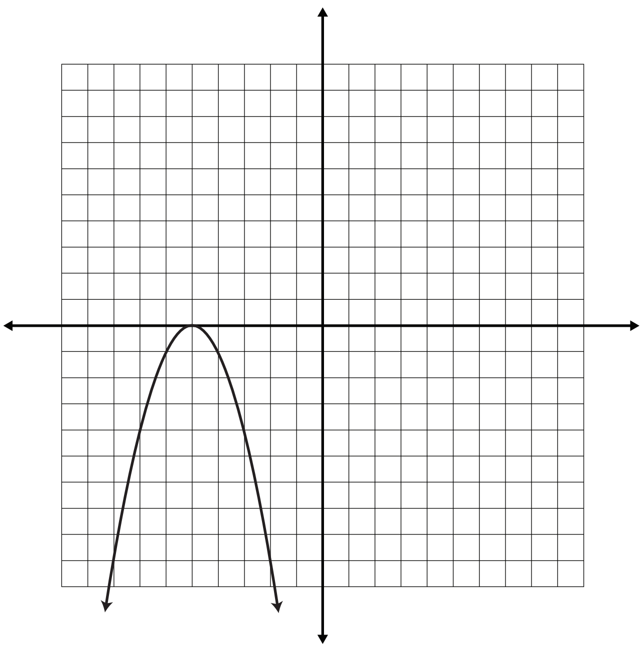
Lesson Notes

Example 8

Given the graph of $y = f(x)$, graph $y = \sqrt{f(x)}$ on the same grid.

Square Root of an Existing Function

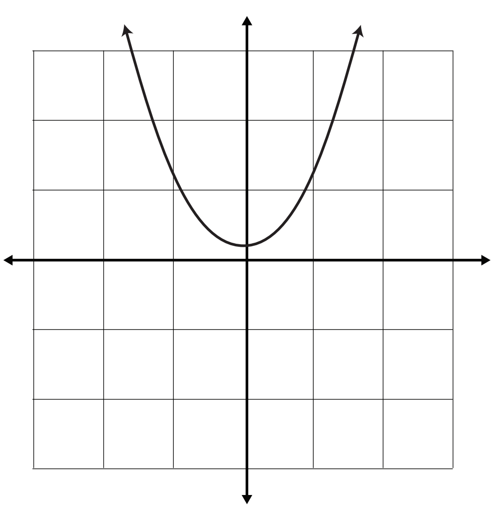
a) $y = -(x + 5)^2$



Domain & Range for $y = f(x)$

Domain & Range for $y = \sqrt{f(x)}$

b) $y = x^2 + 0.25$



Domain & Range for $y = f(x)$

Domain & Range for $y = \sqrt{f(x)}$

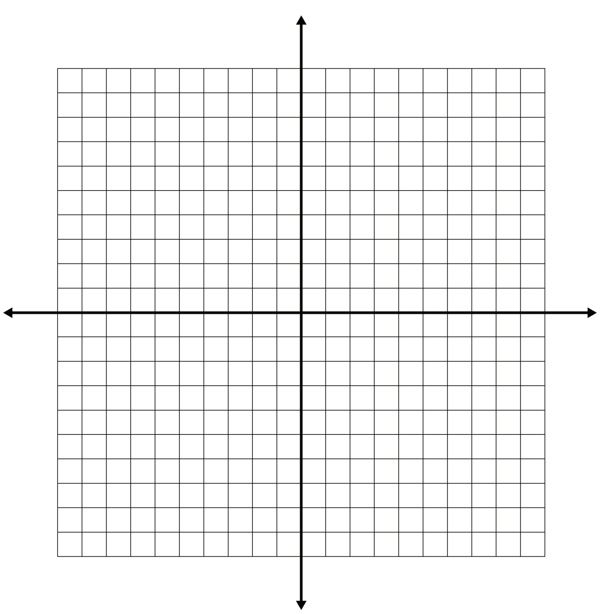
Example 9

Solve the radical equation $\sqrt{x + 2} = 3$ in three different ways.

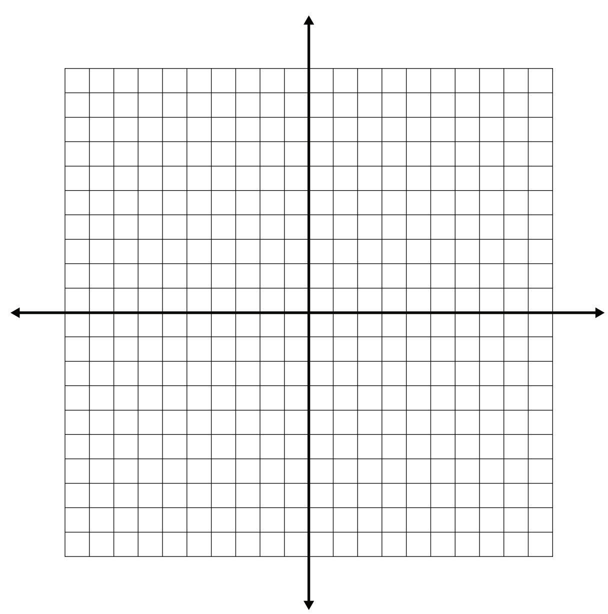
Radical Equations

a) Solve algebraically and check for extraneous roots.

b) Solve by finding the point of intersection of a system of equations.



c) Solve by finding the x-intercept(s) of a single function.



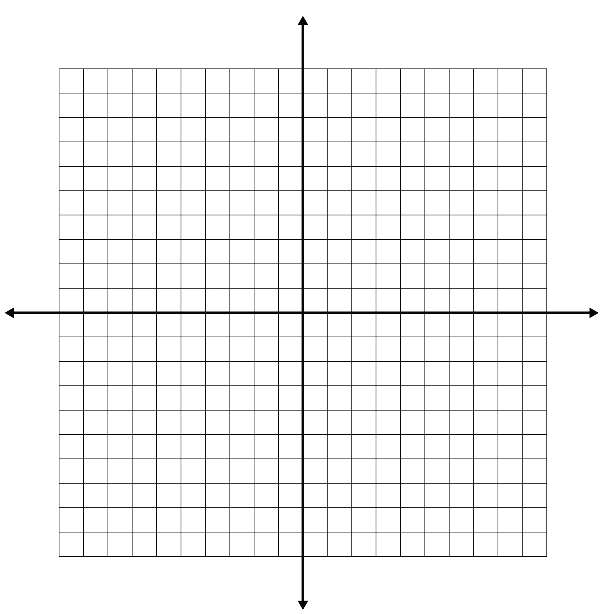
Example 10

Solve the radical equation $x = \sqrt{x + 2}$ in three different ways.

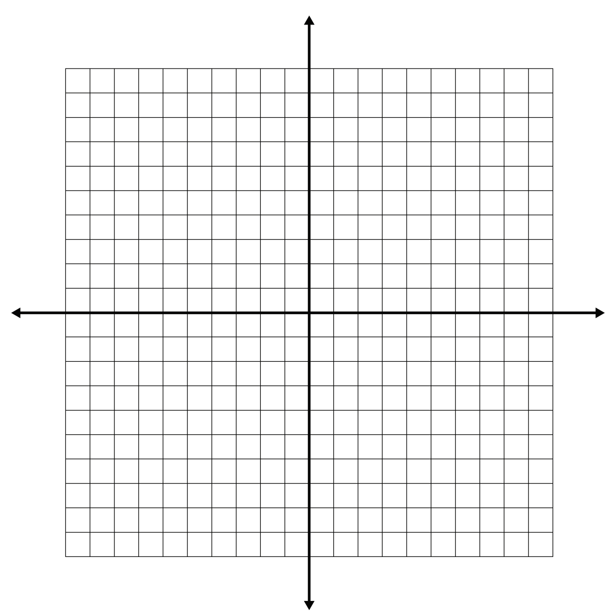
Radical Equations

a) Solve algebraically and check for extraneous roots.

b) Solve by finding the point of intersection of a system of equations.



c) Solve by finding the x-intercept(s) of a single function.

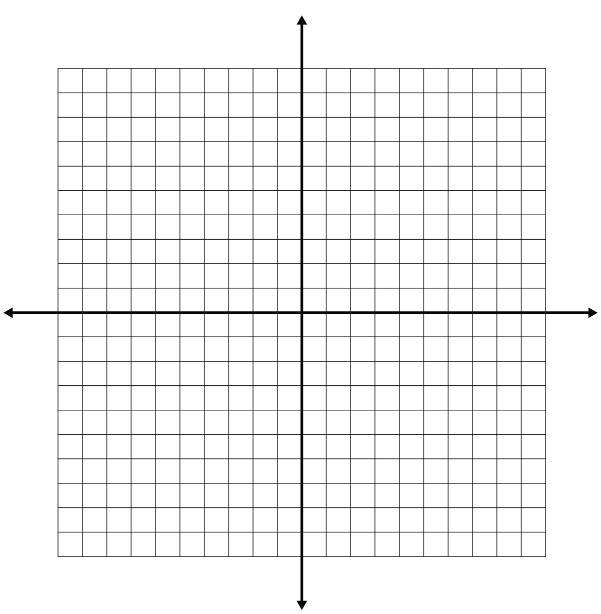


Example 11 Solve the radical equation $2\sqrt{x+3} = x + 3$ in three different ways.

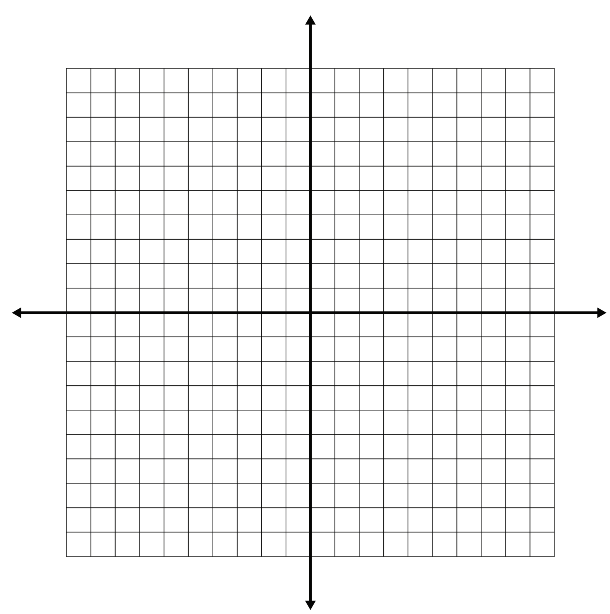
Radical Equations

a) Solve algebraically and check for extraneous roots.

b) Solve by finding the point of intersection of a system of equations.



c) Solve by finding the x-intercept(s) of a single function.



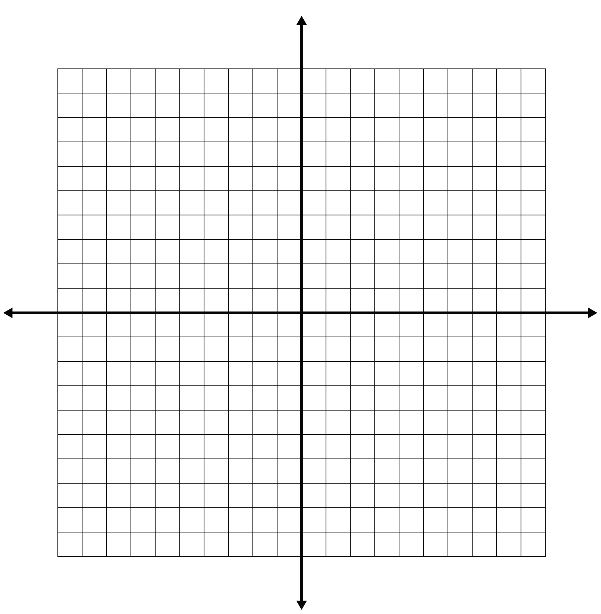
Example 12

Solve the radical equation $\sqrt{16 - x^2} = 5$ in three different ways.

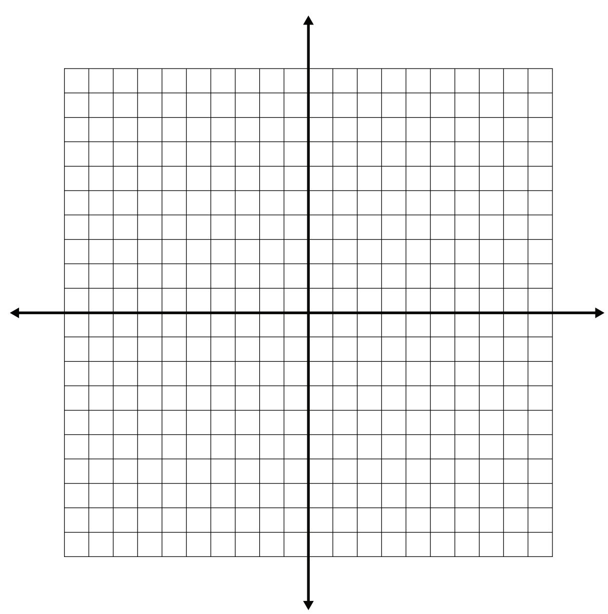
Radical Equations

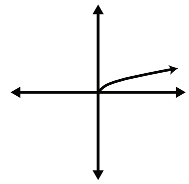
a) Solve algebraically and check for extraneous roots.

b) Solve by finding the point of intersection of a system of equations.



c) Solve by finding the x-intercept(s) of a single function.





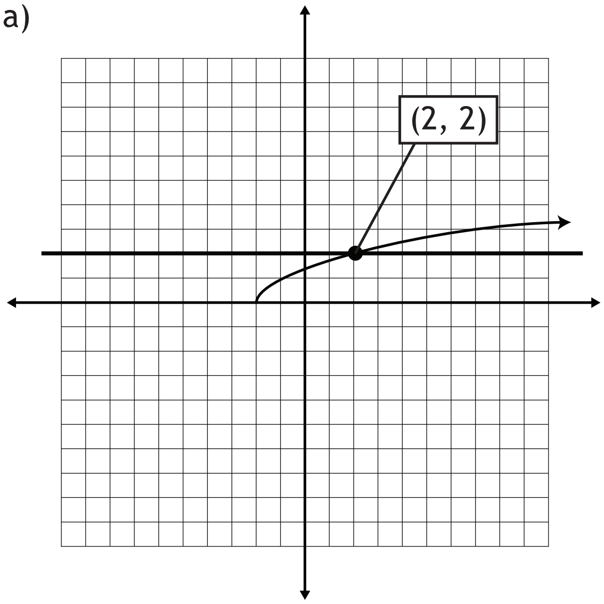
$y = \sqrt{x}$

Example 13

Write an equation that can be used to find the point of intersection for each pair of graphs.

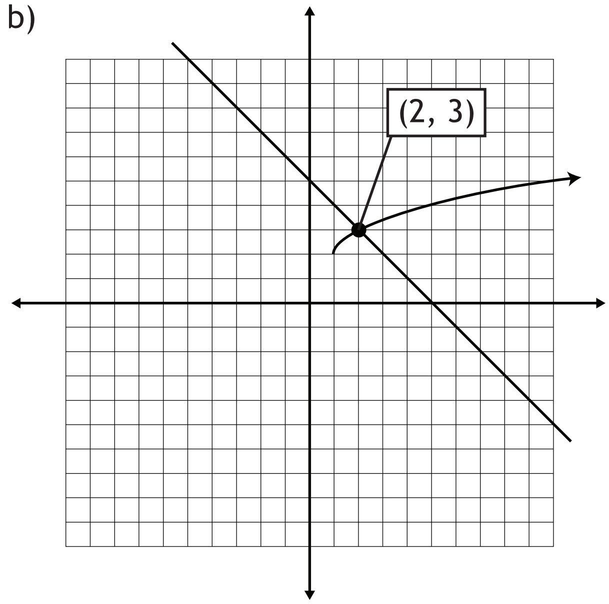
Find the Radical Equation

a)



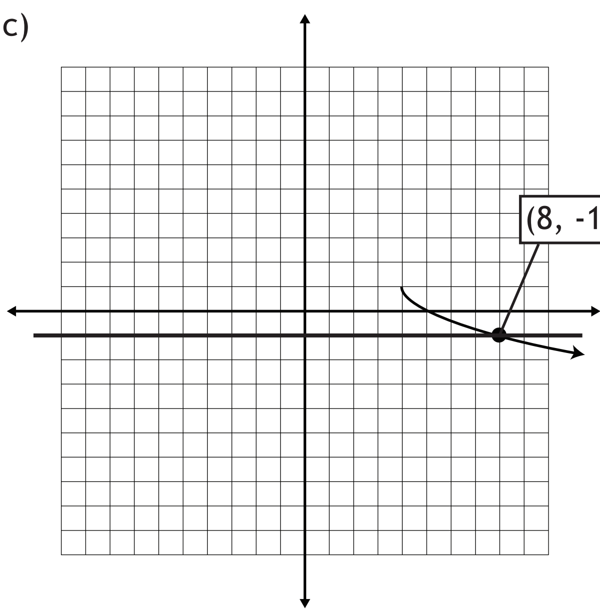
Equation:

b)



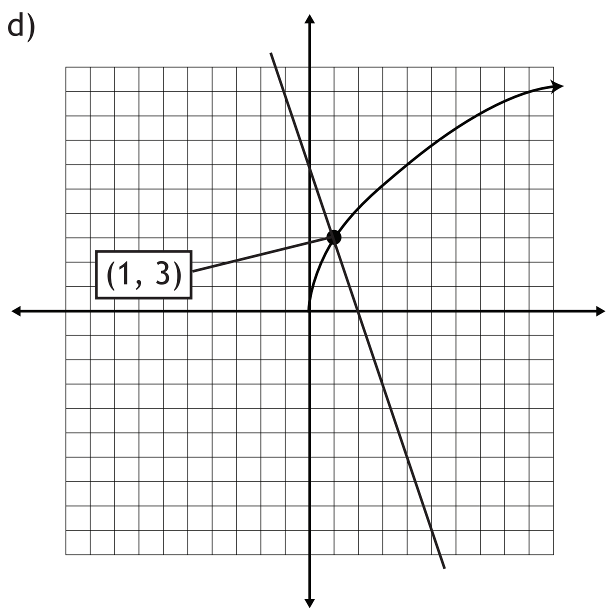
Equation:

c)



Equation:

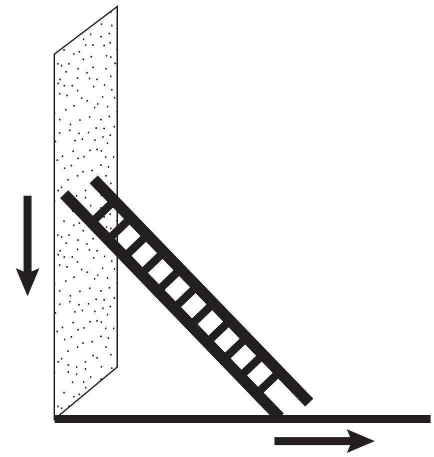
d)



Equation:

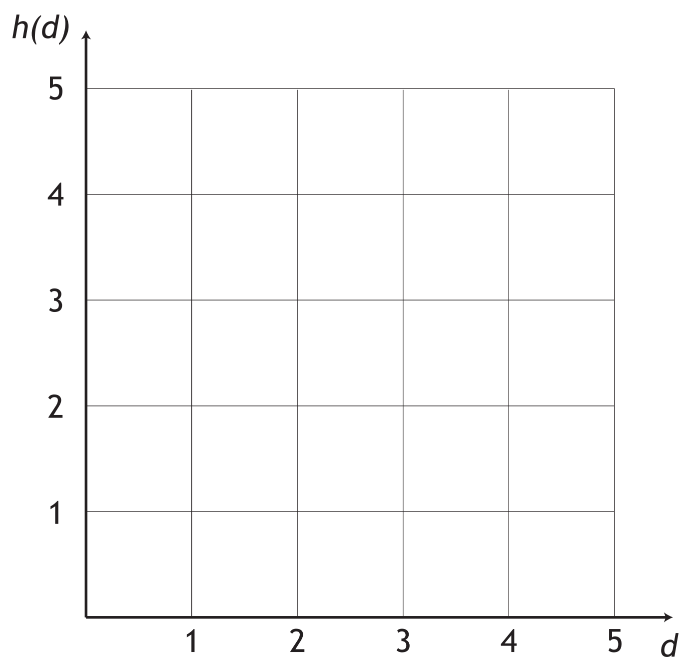
Example 14

A ladder that is 3 m long is leaning against a wall. The base of the ladder is *d* metres from the wall, and the top of the ladder is *h* metres above the ground.

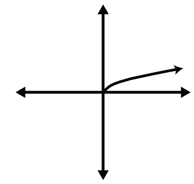


a) Write a function, *h(d)*, to represent the height of the ladder as a function of its base distance *d*.

b) Graph the function and state the domain and range. Describe the ladder's orientation when d = 0 and d = 3.

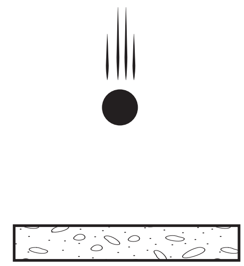


c) How far is the base of the ladder from the wall when the top of the ladder is $\sqrt{5}$ metres above the ground?



$y = \sqrt{x}$

Example 15



If a ball at a height of h metres is dropped,
the length of time it takes to hit the ground is:

$$t = \sqrt{\frac{h}{4.9}}$$

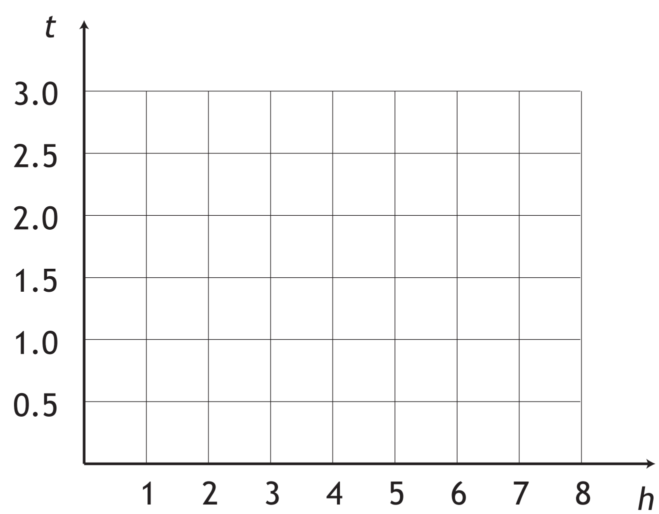
where t is the time in seconds.

a) If a ball is dropped from twice its original height, how will that change the time it takes to fall?

b) If a ball is dropped from one-quarter of its original height,
how will that change the time it takes to fall?

c) The original height of the ball is 4 m. Complete the table of values and draw the graph.
Do your results match the predictions made in parts (a & b)?

h *metres*	***t*** *seconds*
1 *quarter*	
4 *original*	
8 *double*	

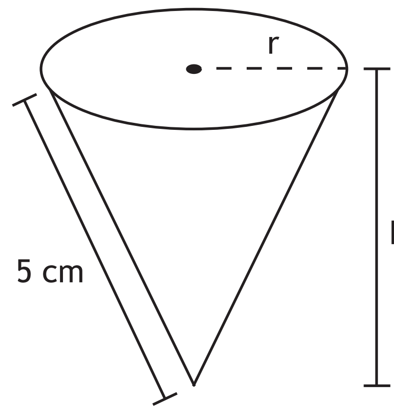


Example 16

A disposable paper cup has the shape of a cone. The volume of the cone is V (cm^3), the radius is r (cm), the height is h (cm), and the slant height is 5 cm.

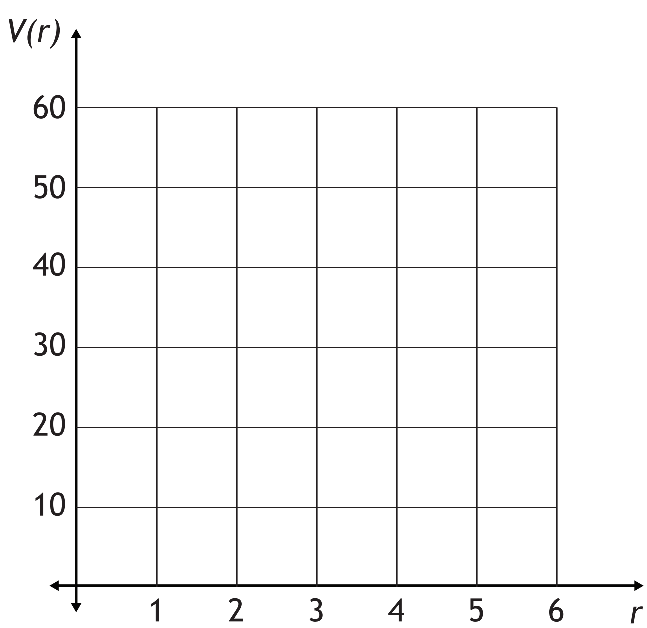
Cone Volume

$$V = \frac{1}{3}\pi r^2 h$$



a) Derive a function, *V(r)*, that expresses the volume of the paper cup as a function of *r*.

b) Graph the function from part (a) and explain the shape of the graph.



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$y = \frac{1}{x}$

Lesson Notes

Example 1

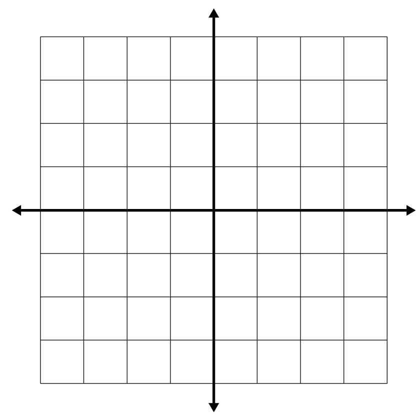
Reciprocal of a ***Linear Function***.

Reciprocal of a Linear Function

a) Fill in the table of values for the function $y = \frac{1}{x}$.

x	*y*
-2	
-1	
-0.5	
-0.25	
0	
0.25	
0.5	
1	
2	

b) Draw the graph of the function $y = \frac{1}{x}$. State the domain and range.



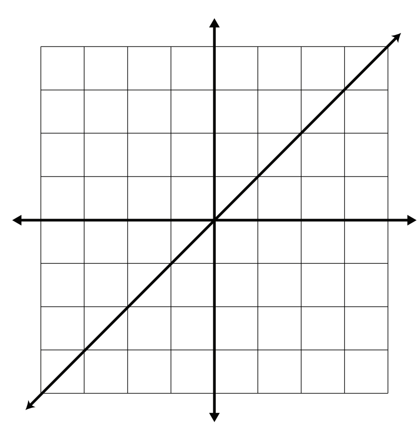
c) Draw the graph of y = x in the same grid used for part (b).
Compare the graph of y = x to the graph of $y = \frac{1}{x}$.

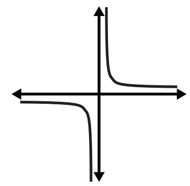
d) Outline a series of steps that can be used to draw the graph of $y = \frac{1}{x}$, starting from y = x.

Step One:

Step Two:

Step Three:



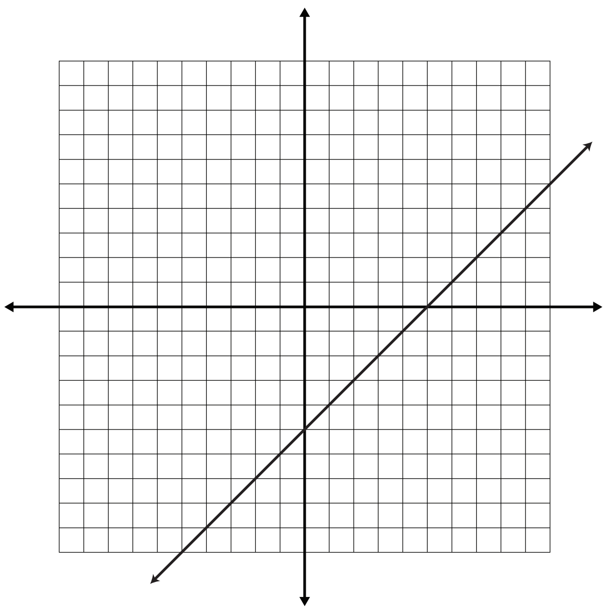


$y = \frac{1}{x}$

Example 2 Given the graph of y = f(x), draw the graph of $y = \frac{1}{f(x)}$.

Reciprocal of a Linear Function

a) y = x - 5

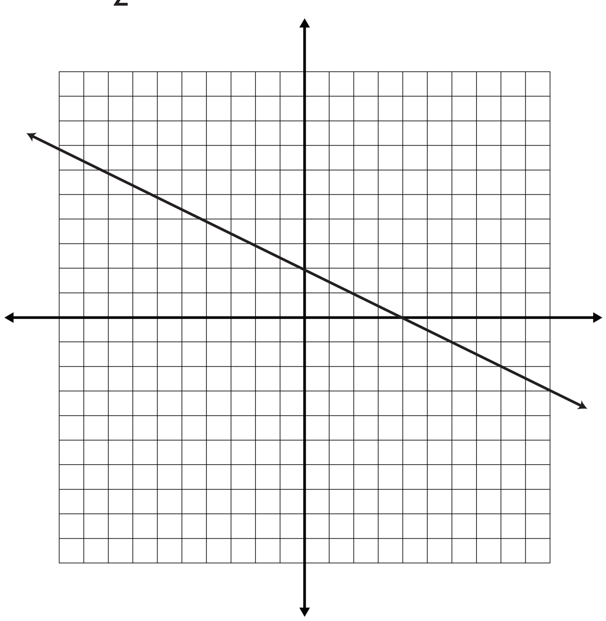


Domain & Range of y = f(x)

Domain & Range of $y = \frac{1}{f(x)}$

Asymptote Equations:

b) $y = -\frac{1}{2}x + 2$



Domain & Range of y = f(x)

Domain & Range of $y = \frac{1}{f(x)}$

Asymptote Equation(s):

Example 3

Reciprocal of a ***Quadratic Function***.

Reciprocal of a Quadratic Function

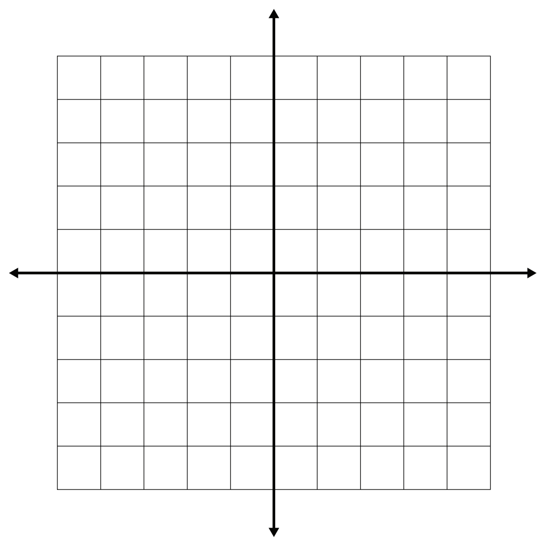
a) Fill in the table of values for the function $y = \frac{1}{x^2 - 4}$.

x	y
-3	
-2	
-1	
0	
1	
2	
3	

x	y
-2.05	
-1.95	

x	y
1.95	
2.05	

b) Draw the graph of the function $y = \frac{1}{x^2 - 4}$. State the domain and range.



c) Draw the graph of $y = x^2 - 4$ in the same grid used for part (b). Compare the graph of $y = x^2 - 4$ to the graph of $y = \frac{1}{x^2 - 4}$.

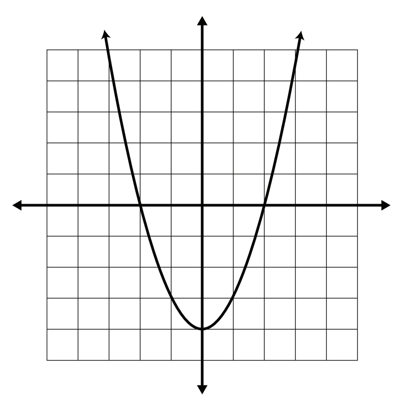
d) Outline a series of steps that can be used to draw the graph of $y = \frac{1}{x^2 - 4}$, starting from $y = x^2 - 4$.

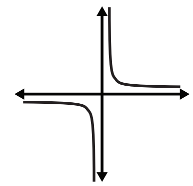
Step One:

Step Two:

Step Three:

Step Four:





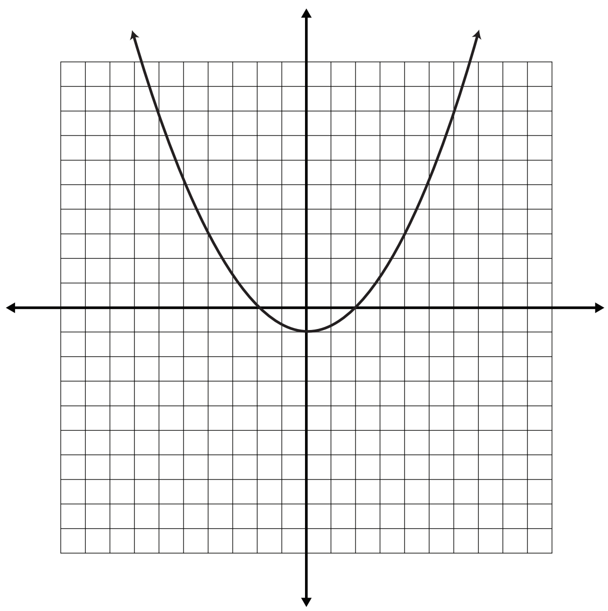
$y = \frac{1}{x}$

Example 4

Given the graph of $y = f(x)$, draw the graph of $y = \frac{1}{f(x)}$.

Reciprocal of a Quadratic Function

a) $y = \frac{1}{4}x^2 - 1$

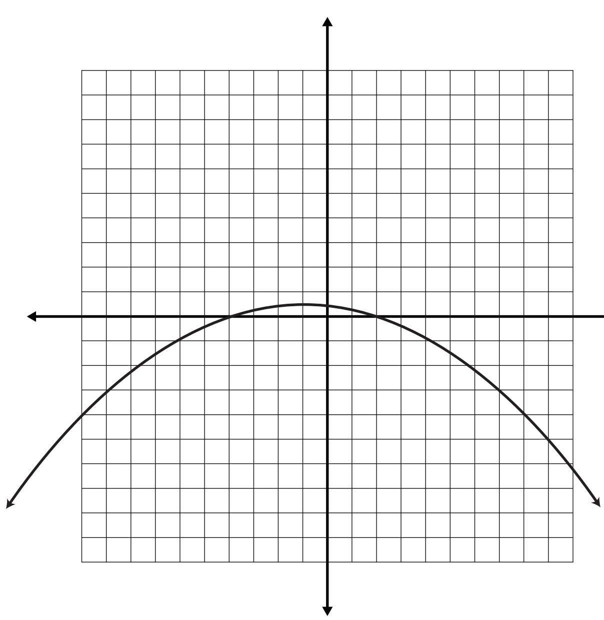


Domain & Range of $y = f(x)$

Domain & Range of $y = \frac{1}{f(x)}$

Asymptote Equation(s):

b) $y = -\frac{1}{18}(x + 1)^2 + \frac{1}{2}$

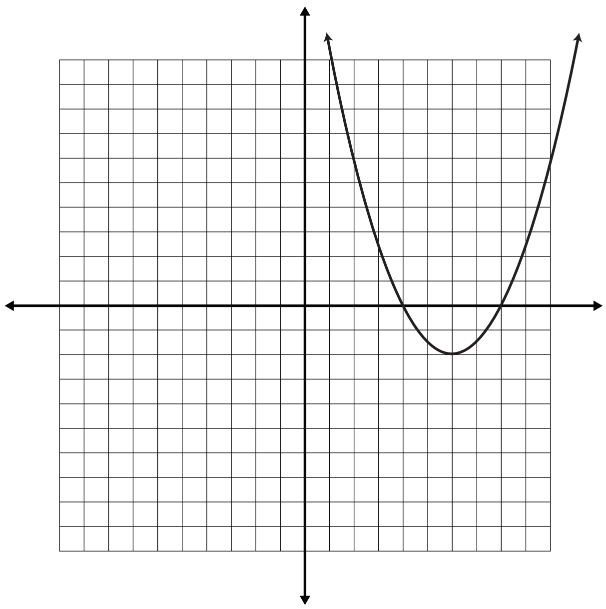


Domain & Range of $y = f(x)$

Domain & Range of $y = \frac{1}{f(x)}$

Asymptote Equation(s):

c) $y = \frac{1}{2}(x - 6)^2 - 2$

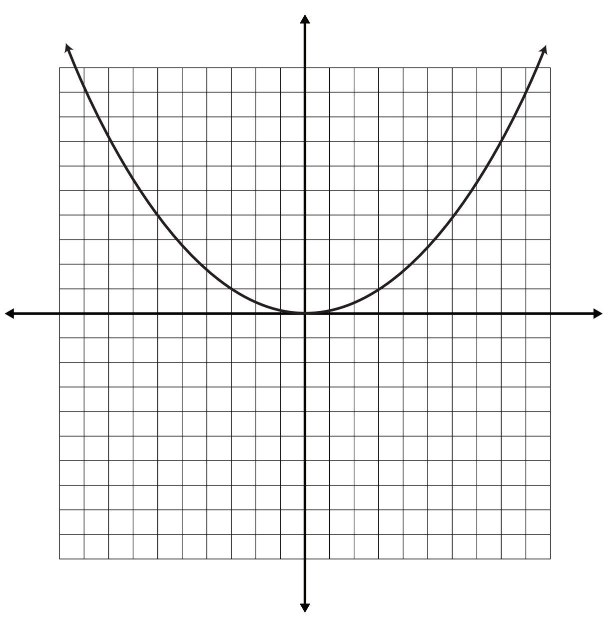


Domain & Range of $y = f(x)$

Domain & Range of $y = \frac{1}{f(x)}$

Asymptote Equation(s):

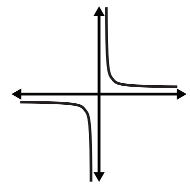
d) $y = \frac{1}{9}x^2$



Domain & Range of $y = f(x)$

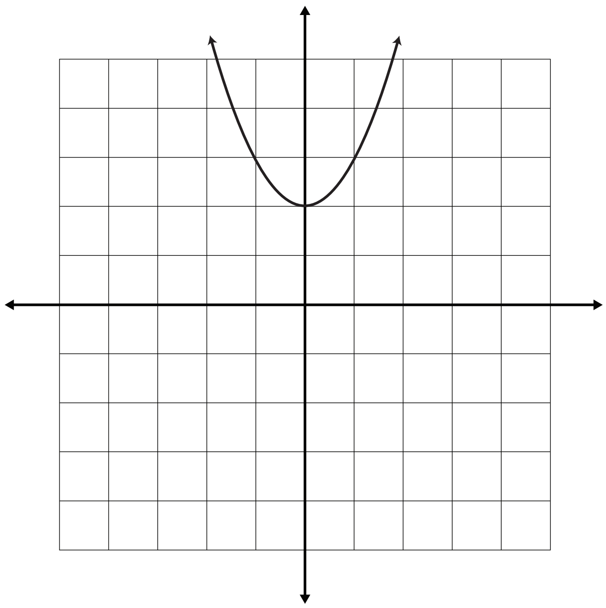
Domain & Range of $y = \frac{1}{f(x)}$

Asymptote Equation(s):



$y = \frac{1}{x}$

e) $y = x^2 + 2$

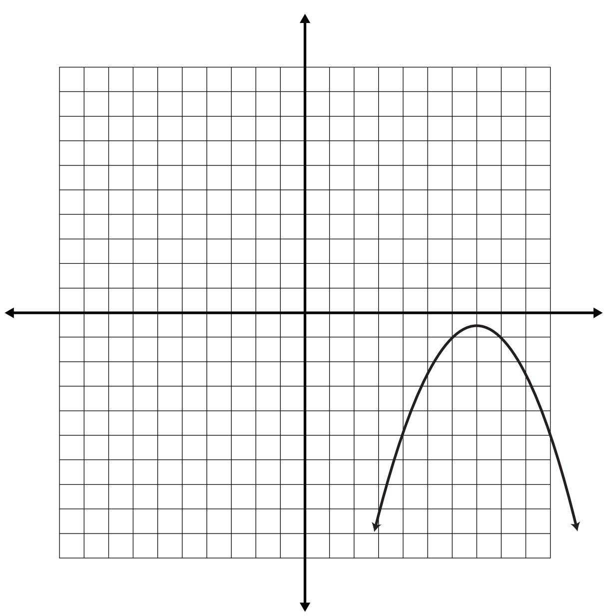


Domain & Range of y = f(x)

Domain & Range of $y = \frac{1}{f(x)}$

Asymptote Equation(s):

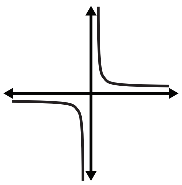
f) $y = -\frac{1}{2}(x - 7)^2 - \frac{1}{2}$



Domain & Range of y = f(x)

Domain & Range of $y = \frac{1}{f(x)}$

Asymptote Equation(s):



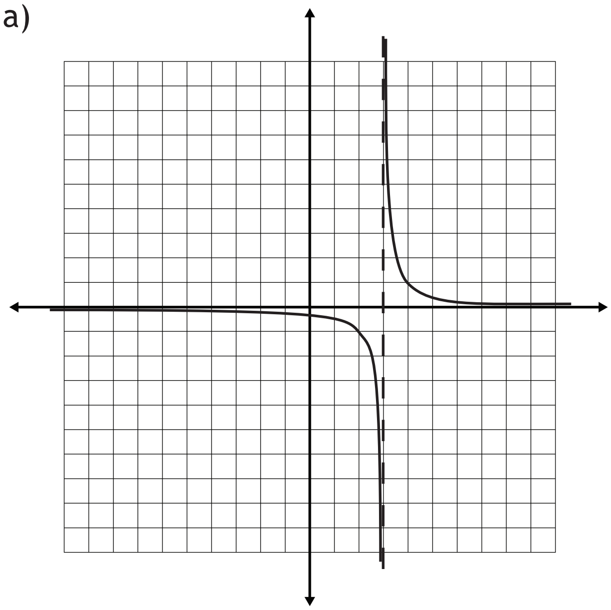
$y = \frac{1}{x}$

Example 5

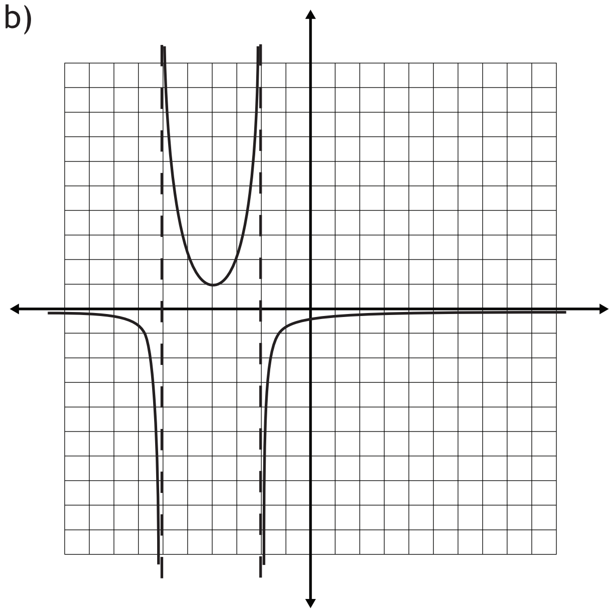
Given the graph of $y = \frac{1}{f(x)}$, draw the graph of $y = f(x)$.

Find the Original Function

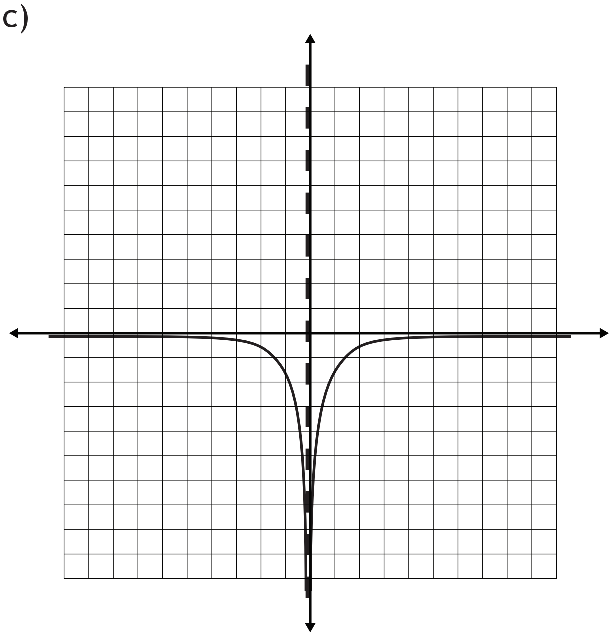
a)



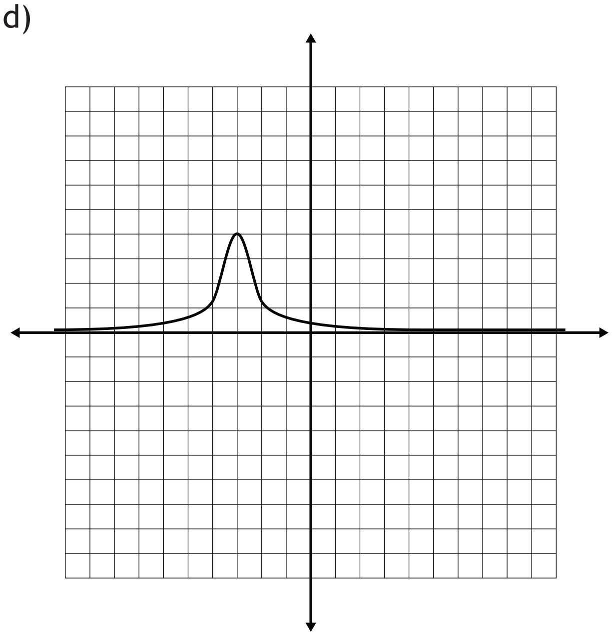
b)

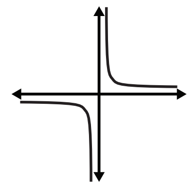


c)



d)





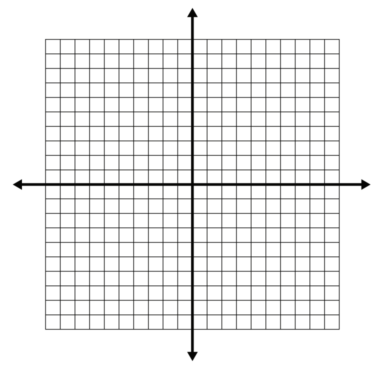
$y = \frac{1}{x}$

Example 6

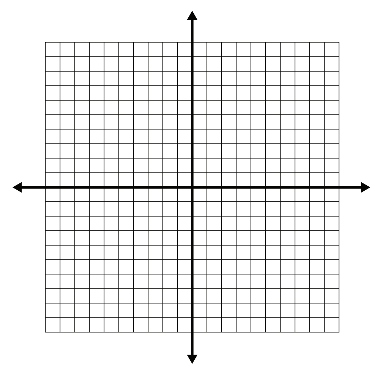
For each function, determine the equations of all asymptotes. Check with a graphing calculator.

Asymptote Equations

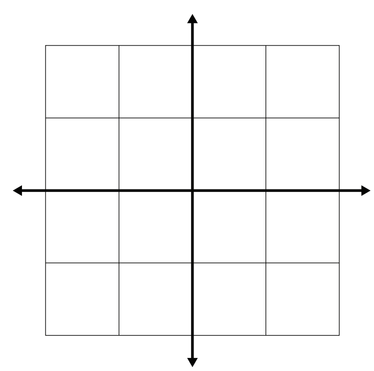
a) $f(x) = \frac{1}{2x - 3}$



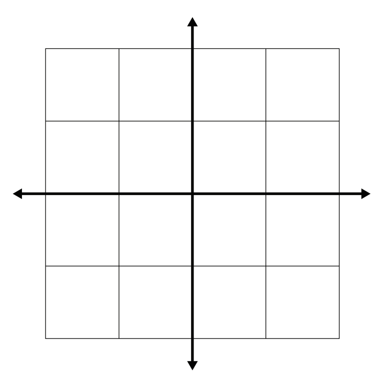
b) $f(x) = \frac{1}{x^2 - 2x - 24}$



c) $f(x) = \frac{1}{6x^3 - 5x^2 - 4x}$



d) $f(x) = \frac{1}{4x^2 + 9}$



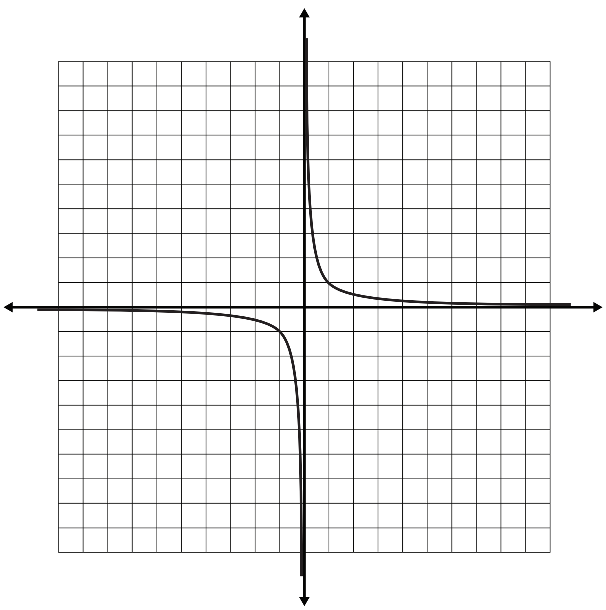
Example 7

Compare each of the following functions to $y = 1/x$ by identifying any stretches or translations, then draw the graph without using technology.

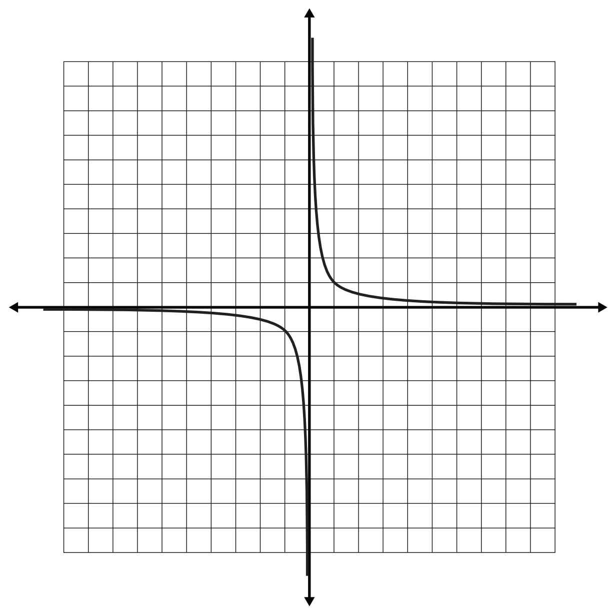
Transformations of Reciprocal Functions

The graph of $y = 1/x$ is provided as a convenience.

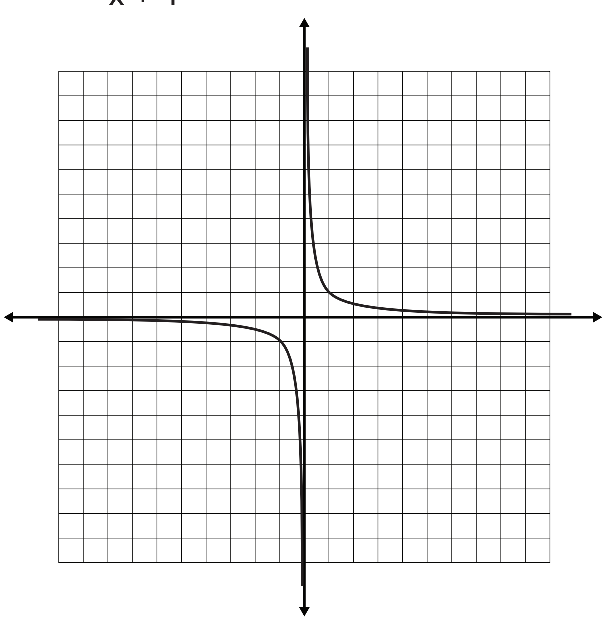
a) $y = \frac{4}{x}$



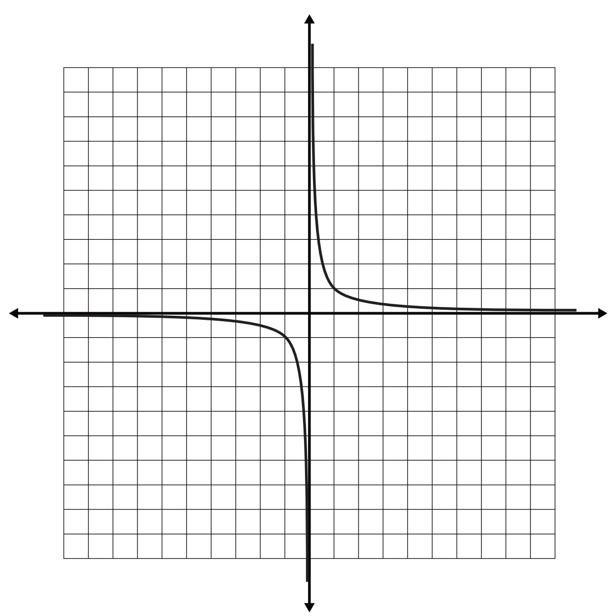
b) $y = \frac{1}{x} - 3$

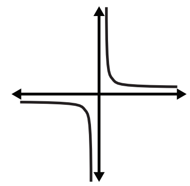


c) $y = \frac{3}{x + 4}$



d) $y = \frac{2}{x - 3} + 2$





$y = \frac{1}{x}$

Example 8

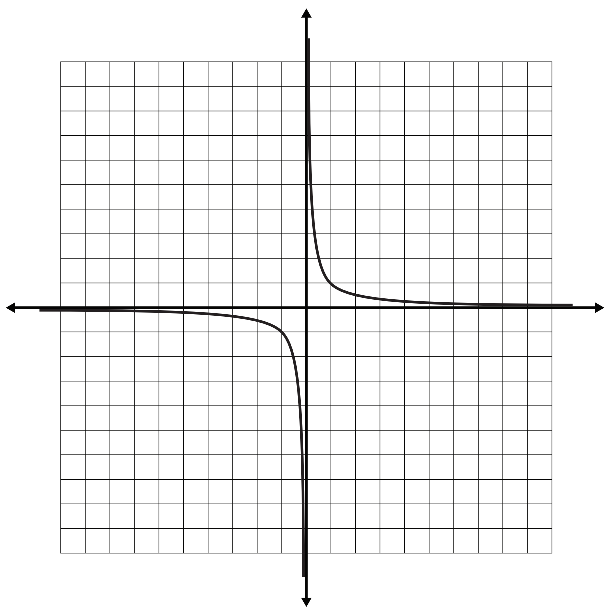
Convert each of the following functions to the form $y = a\left(\frac{1}{x-h}\right) + k$.

Identify the stretches and translations, then draw the graph without using technology.

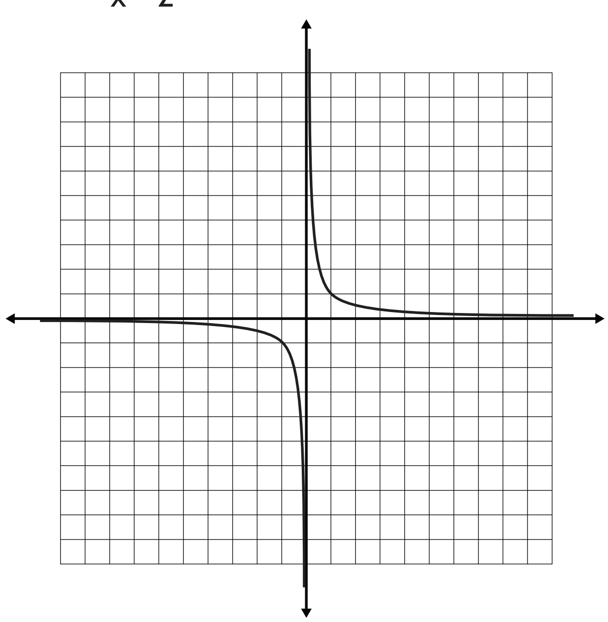
Transformations of Reciprocal Functions

The graph of y = 1/x is provided as a convenience.

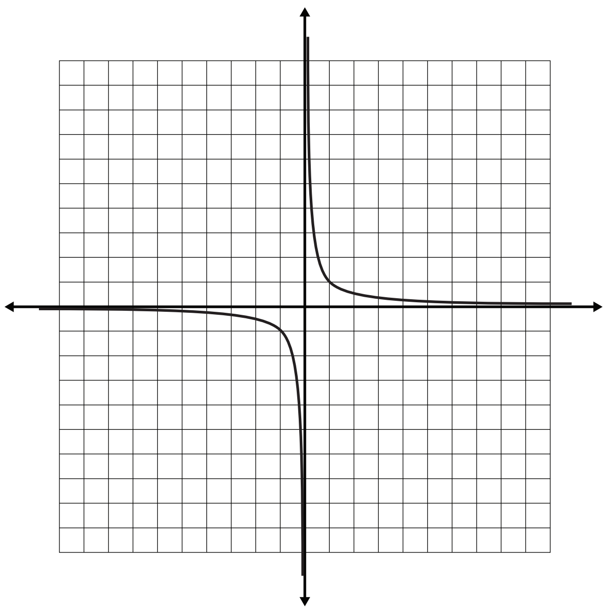
a) $y = \frac{1 - 2x}{x}$



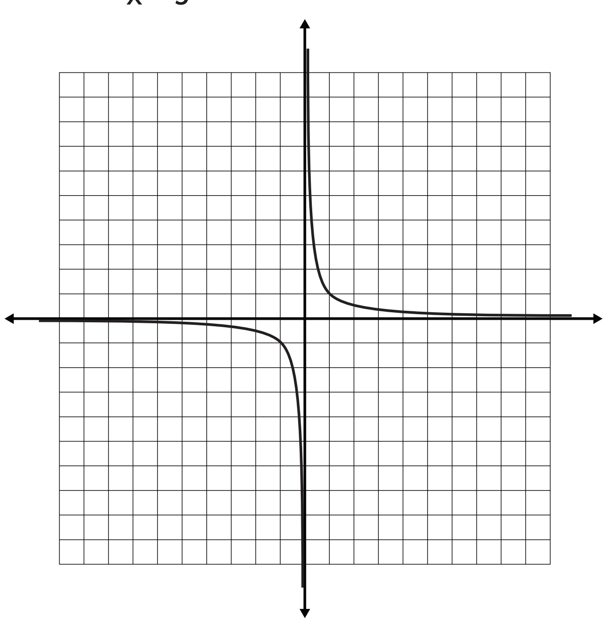
b) $y = \frac{x - 1}{x - 2}$

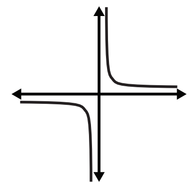


c) $y = \frac{6 - 2x}{x - 1}$



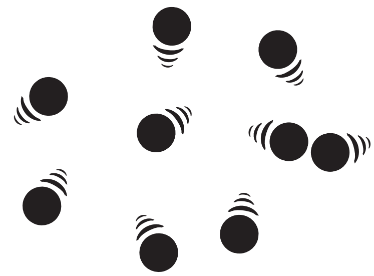
d) $y = \frac{33 - 6x}{x - 5}$





$y = \frac{1}{x}$

Example 9 *Chemistry Application: Ideal Gas Law*



The ideal gas law relates the pressure, volume, temperature, and molar amount of a gas with the formula:

$$PV = nRT$$

where P is the pressure in kilopascals (kPa), V is the volume in litres (L), n is the molar amount of the gas (mol), R is the universal gas constant, and T is the temperature in kelvins (K).

An ideal gas law experiment uses 0.011 mol of a gas at a temperature of 273.15 K.

a) If the temperature and molar amount of the gas are held constant, the ideal gas law follows a reciprocal relationship and can be written as a rational function, P(V). Write this function.

b) If the original volume of the gas is doubled, how will the pressure change?

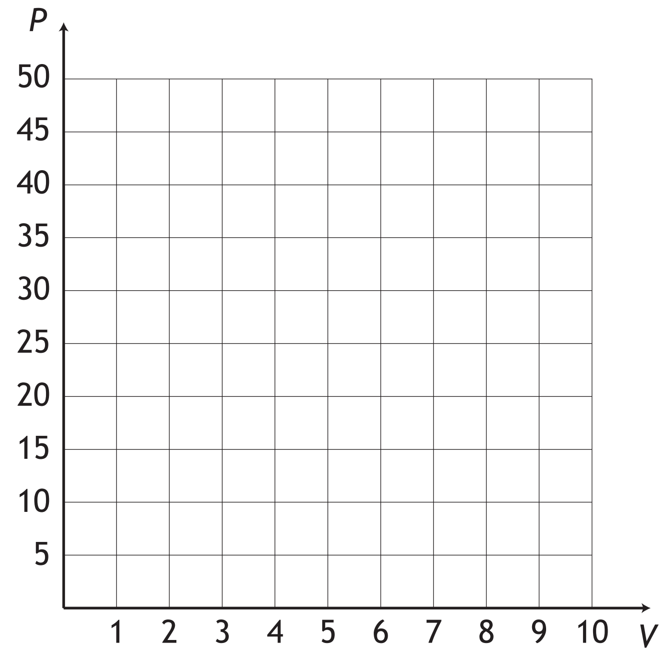
c) If the original volume of the gas is halved, how will the pressure change?

d) If P(5.0 L) = 5.0 kPa, determine the experimental value of the universal gas constant R.

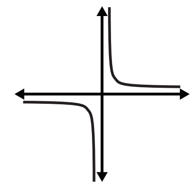
e) Complete the table of values and draw the graph for this experiment.

V (L)	***P*** (kPa)
0.5	
1.0	
2.0	
5.0	
10.0	

Pressure V.S. Volume of 0.011 mol of a gas at 273.15 K

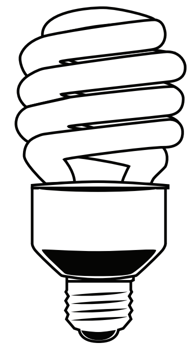


f) Do the results from the table match the predictions in parts b & c?



$y = \frac{1}{x}$

Example 10 ***Physics Application: Light Illuminance***



Objects close to a light source appear brighter than objects farther away. This phenomenon is due to the *illuminance* of light, a measure of how much light is incident on a surface. The illuminance of light can be described with the reciprocal-square relation:

$$I(d) = \frac{S}{4\pi d^2}$$

where I is the illuminance (SI unit = lux), S is the amount of light emitted by a source (SI unit = lumens), and d is the distance from the light source in metres.

In an experiment to investigate the reciprocal-square nature of light illuminance, a screen can be moved from a baseline position to various distances from the bulb.

a) If the original distance of the screen from the bulb is doubled, how does the illuminance change?

b) If the original distance of the screen from the bulb is tripled, how does the illuminance change?

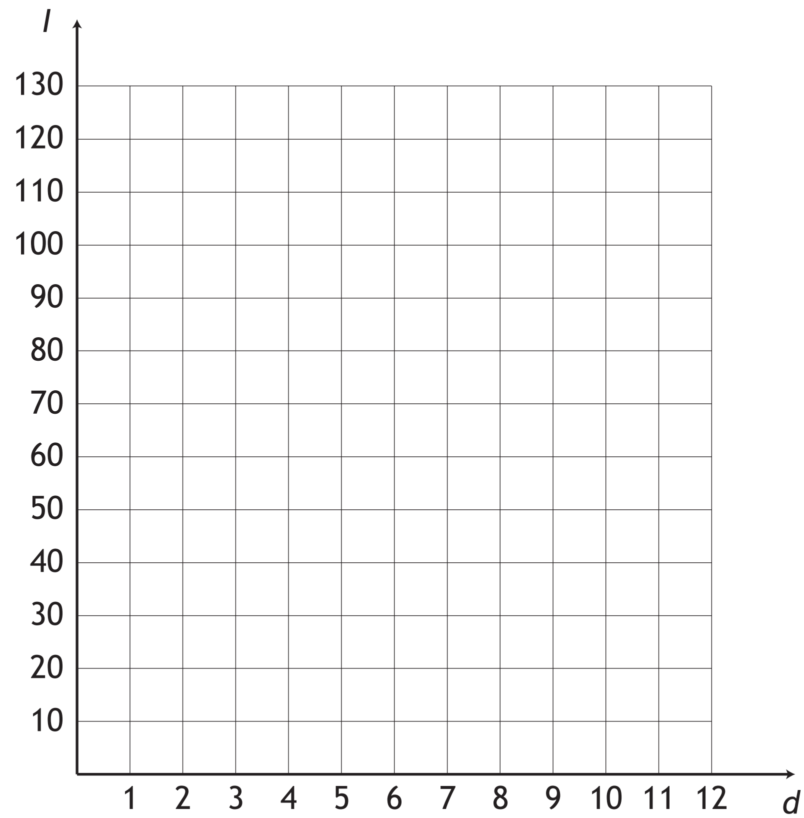
c) If the original distance of the screen from the bulb is halved, how does the illuminance change?

d) If the original distance of the screen from the bulb is quartered, how does the illuminance change?

e) A typical household fluorescent bulb emits 1600 lumens. If the original distance from the bulb to the screen was 4 m, complete the table of values and draw the graph.

d (m)	***I*** (W/m²)
1	
2	
4 ORIGINAL	
8	
12	

Illuminance V.S. Distance for a Fluorescent Bulb



f) Do the results from the table match the predictions made in parts a-d?

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Lesson Notes

Example 1

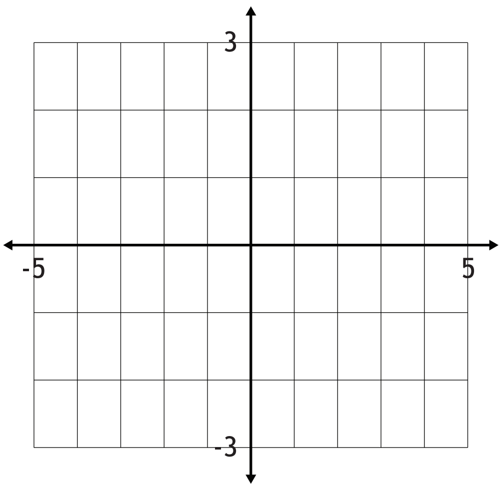
Numerator Degree < Denominator Degree

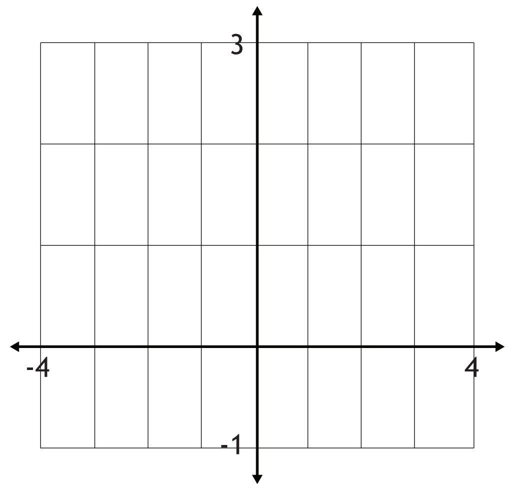
$\dfrac{\text{Numerator}}{\text{Denominator}}$

Predict if any asymptotes or holes are present in the graph of each rational function. Use a graphing calculator to draw the graph and verify your prediction.

a) $y = \dfrac{x}{x^2 - 9}$

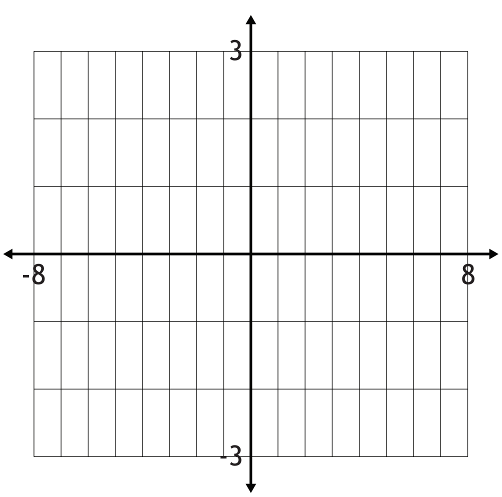
b) $y = \dfrac{x + 2}{x^2 + 1}$

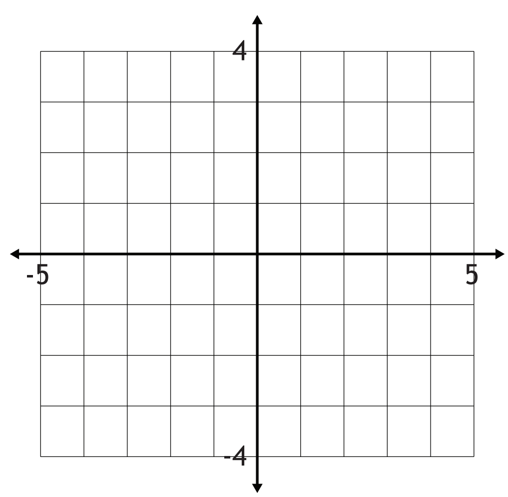




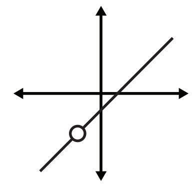
c) $y = \dfrac{x + 4}{x^2 - 16}$

d) $y = \dfrac{x^2 - x - 2}{x^3 - x^2 - 2x}$





Lesson Notes



$$y = \frac{x^2 + x - 2}{x + 2}$$

Example 2

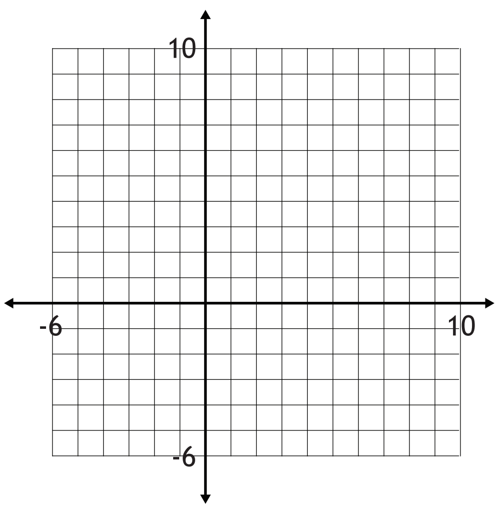
Numerator Degree = Denominator Degree

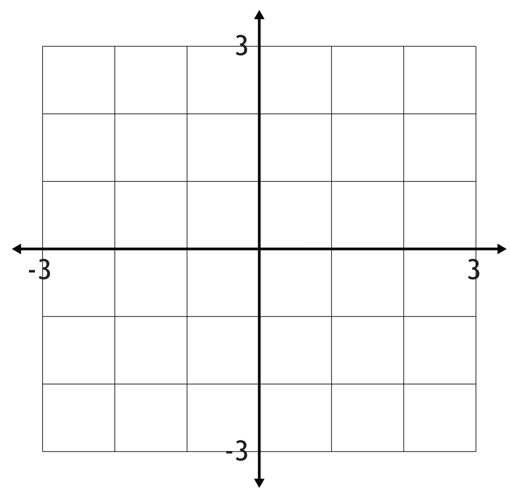
$$\frac{\text{Numerator}}{\text{Denominator}}$$

Predict if any asymptotes or holes are present in the graph of each rational function. Use a graphing calculator to draw the graph and verify your prediction.

a) $y = \frac{4x}{x - 2}$

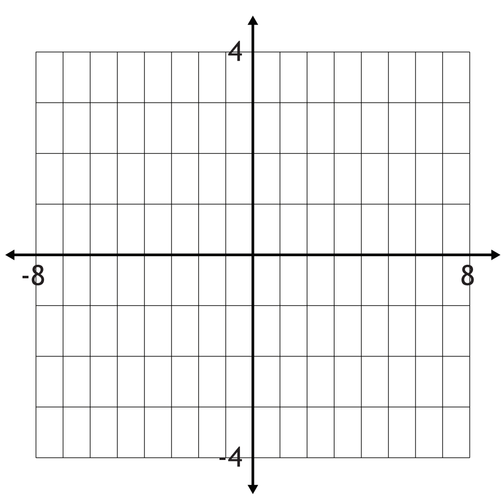
b) $y = \frac{x^2}{x^2 - 1}$

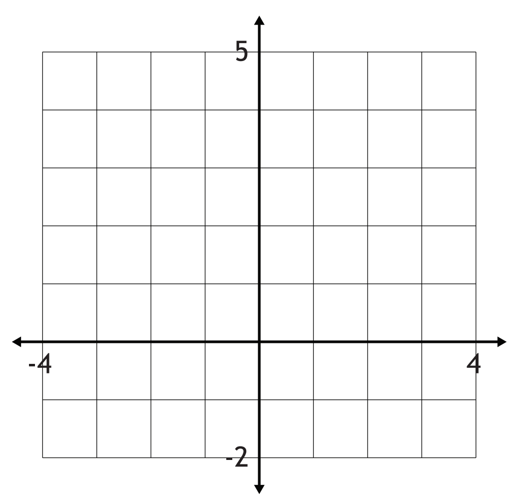


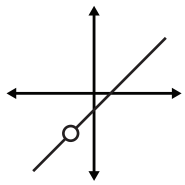


c) $y = \frac{3x^2}{x^2 + 9}$

d) $y = \frac{3x^2 - 3x - 18}{x^2 - x - 6}$







$$y = \frac{x^2 + x - 2}{x + 2}$$

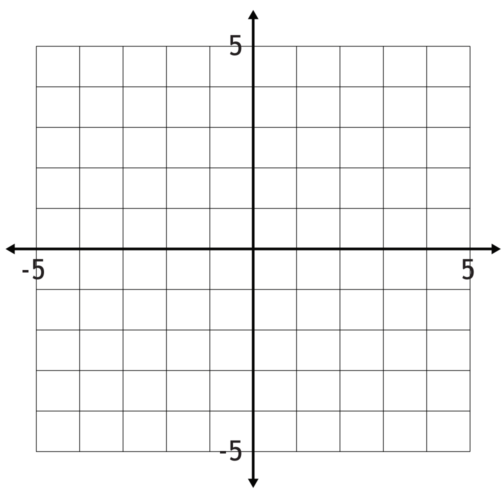
Example 3

Numerator Degree > Denominator Degree

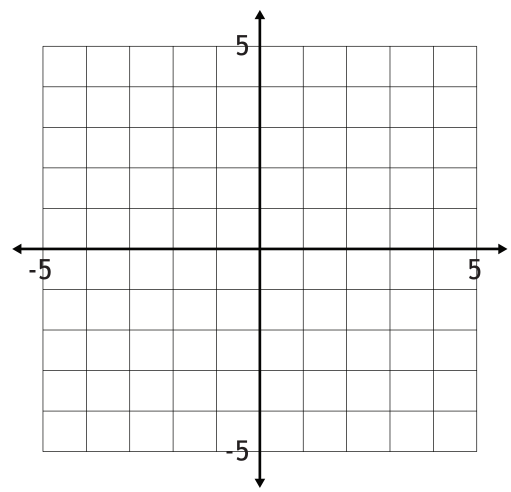
Numerator
Denominator

Predict if any asymptotes or holes are present in the graph of each rational function. Use a graphing calculator to draw the graph and verify your prediction.

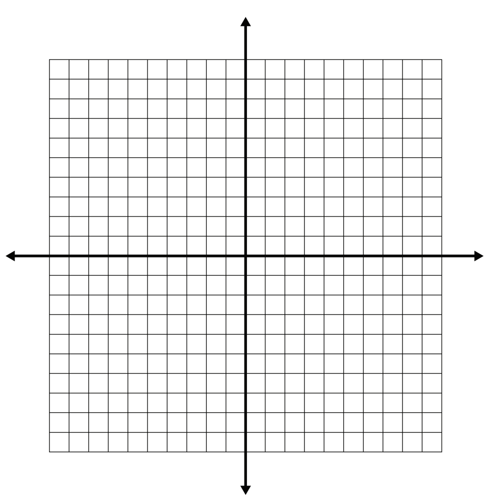
a) $y = \frac{x^2 + 5x + 4}{x + 4}$



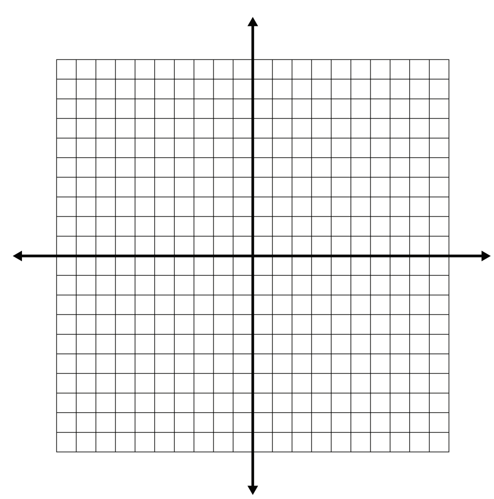
b) $y = \frac{x^2 - 4x + 3}{x - 3}$



c) $y = \frac{x^2 + 5}{x - 1}$



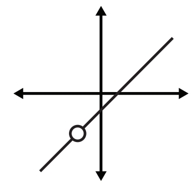
d) $y = \frac{x^2 - x - 6}{x + 1}$



Polynomial, Radical, and Rational Functions

LESSON SIX - *Rational Functions II*

Lesson Notes



$$y = \frac{x^2 + x - 2}{x + 2}$$

Example 4 Graph $y = \dfrac{x}{x^2 - 16}$ without using the graphing feature of your calculator.

Properties of Rational Function Graphs

i) Horizontal Asymptote:

ii) Vertical Asymptote(s):

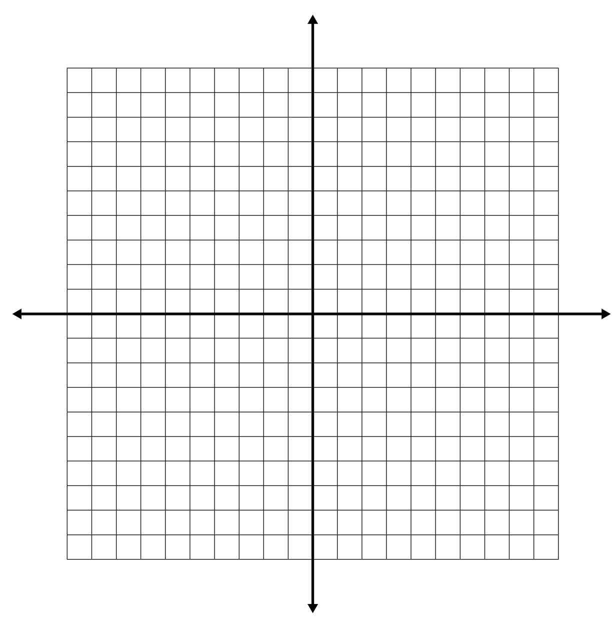
iii) y - intercept:

iv) x - intercept(s):

v) Domain and Range:

Other Points:

These are any extra points required to shape the graph. You may use your calculator to evaluate these.



Example 5

Graph $y = \frac{2x - 6}{x + 2}$ without using the graphing feature of your calculator.

Properties of Rational Function Graphs

i) Horizontal Asymptote:

ii) Vertical Asymptote(s):

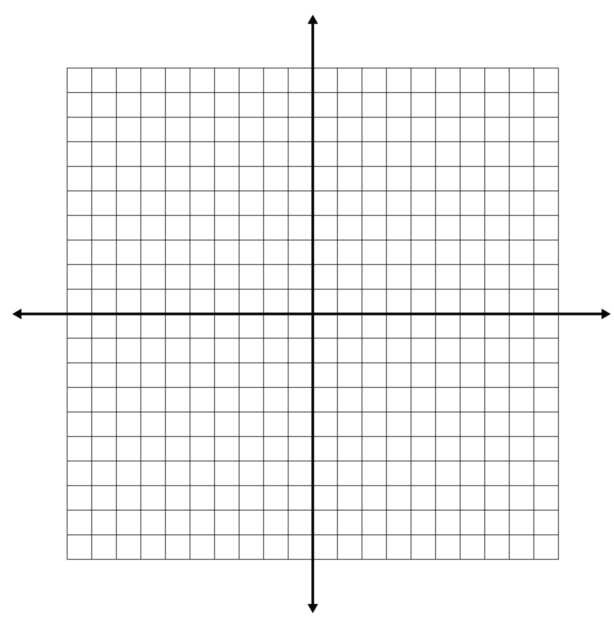
iii) y - intercept:

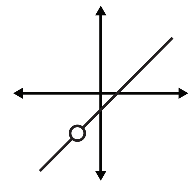
iv) x - intercept(s):

v) Domain and Range:

Other Points:

These are any extra points required to shape the graph. You may use your calculator to evaluate these.





$y = \frac{x^2 + x - 2}{x + 2}$

Example 6 Graph $y = \frac{x^2 + 2x - 8}{x - 1}$ without using the graphing feature of your calculator.

Properties of Rational Function Graphs

i) Horizontal Asymptote:

ii) Vertical Asymptote(s):

iii) y - intercept:

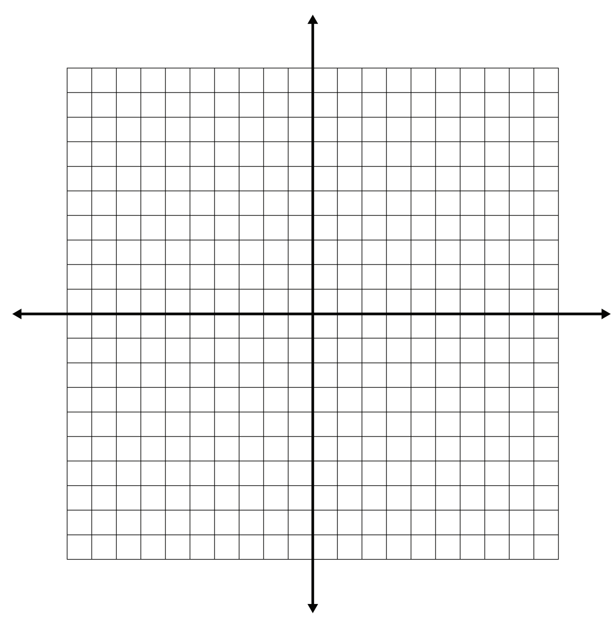
iv) x - intercept(s):

v) Domain and Range:

vi) Oblique Asymptote

Other Points:

These are any extra points required to shape the graph. You may use your calculator to evaluate these.



Example 7

Graph $y = \frac{x^2 - 5x + 6}{x - 2}$ without using the graphing feature of your calculator.

Properties of Rational Function Graphs

i) Can this rational function be simplified?

ii) Holes:

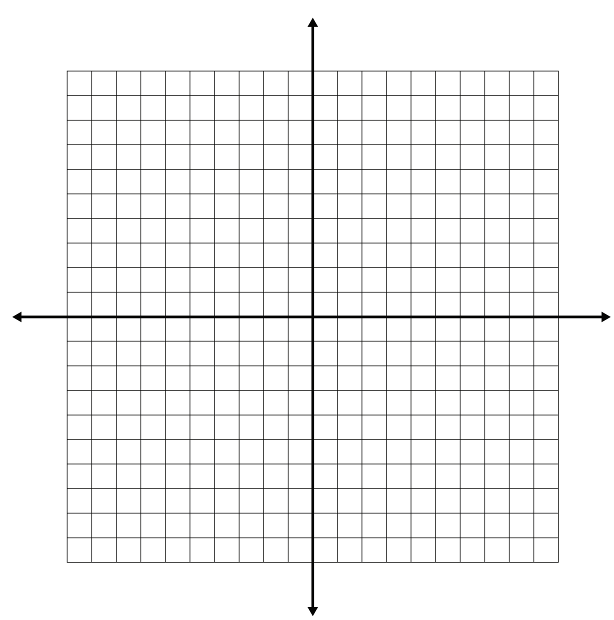
iii) y - intercept:

iv) x - intercept(s):

v) Domain and Range:

Other Points:

These are any extra points required to shape the graph. You may use your calculator to evaluate these.



Polynomial, Radical, and Rational Functions
LESSON SIX - *Rational Functions II*
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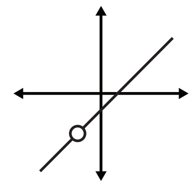


$y = \frac{x^2 + x - 2}{x + 2}$

Example 8

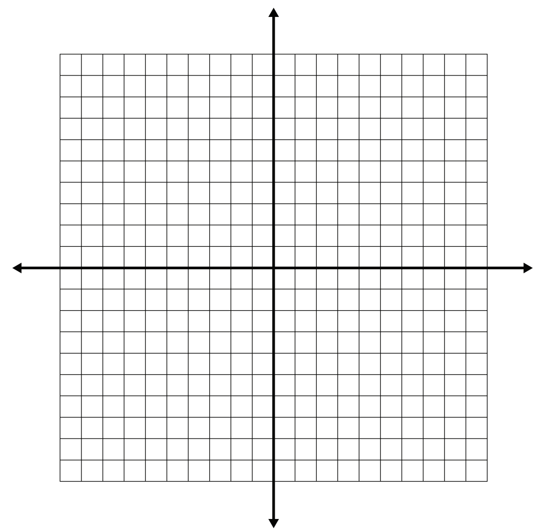
Find the rational function with each set of characteristics and draw the graph.

Finding a Rational Function from its Properties or Graph.

a)

vertical asymptote(s)	x = -2, x = 4
horizontal asymptote	y = 1
x-intercept(s)	(-3, 0) and (5, 0)
hole(s)	none

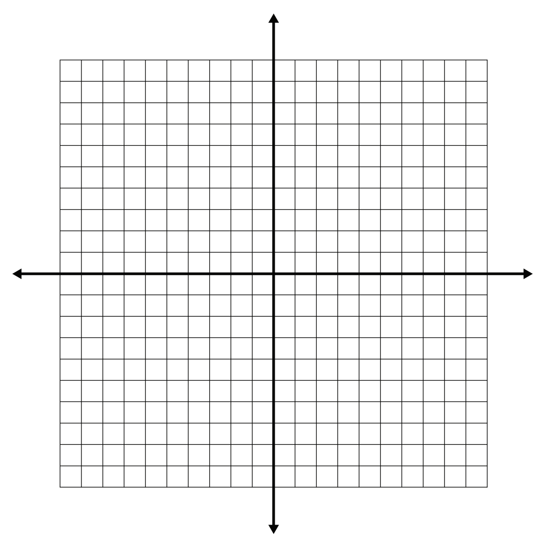
Rational Function:

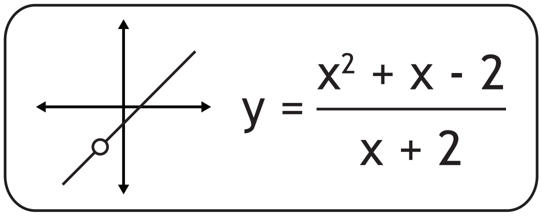


b)

vertical asymptote(s)	x = 0
horizontal asymptote	y = 0
x-intercept(s)	none
hole(s)	(-1, -1)

Rational Function:

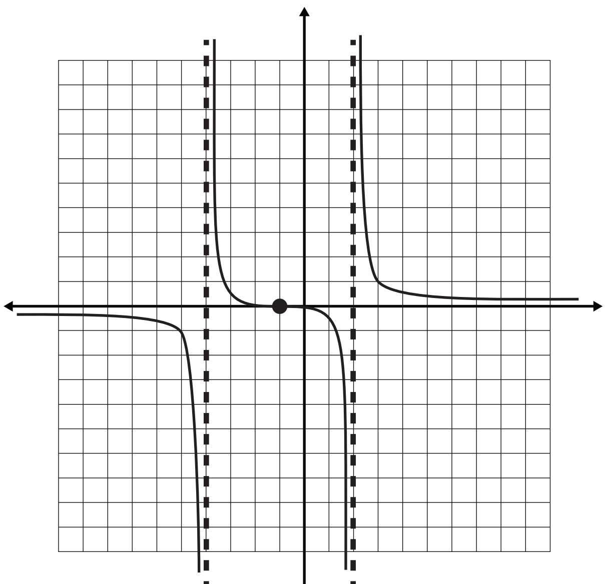




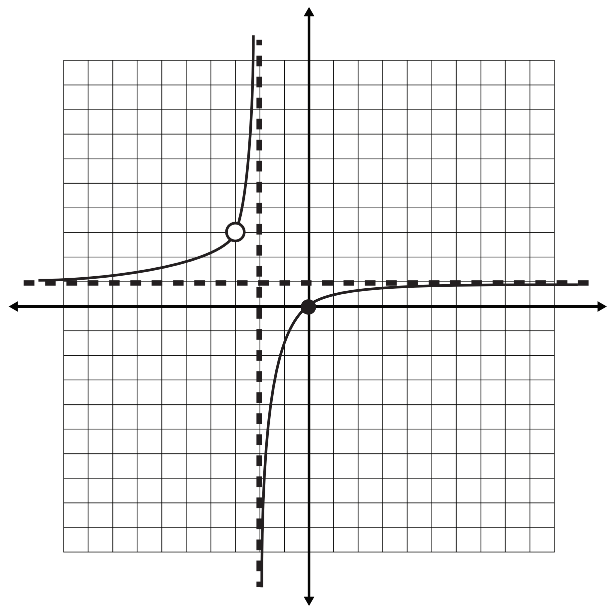
Example 9 Find the rational function shown in each graph.

Finding a Rational Function from its Properties or Graph.

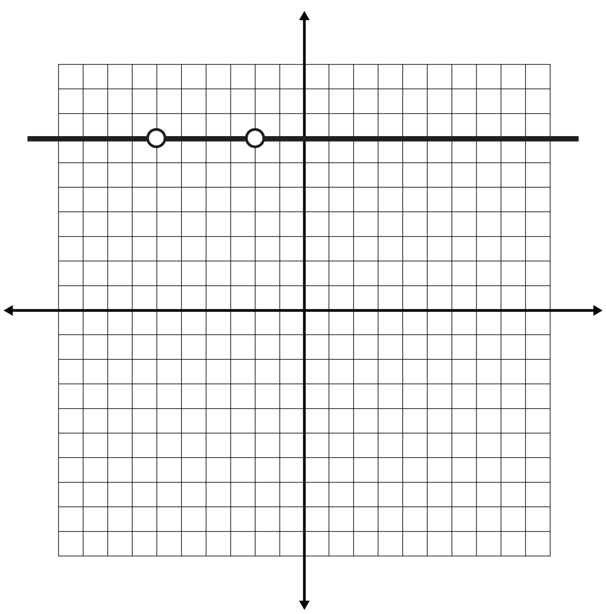
a)



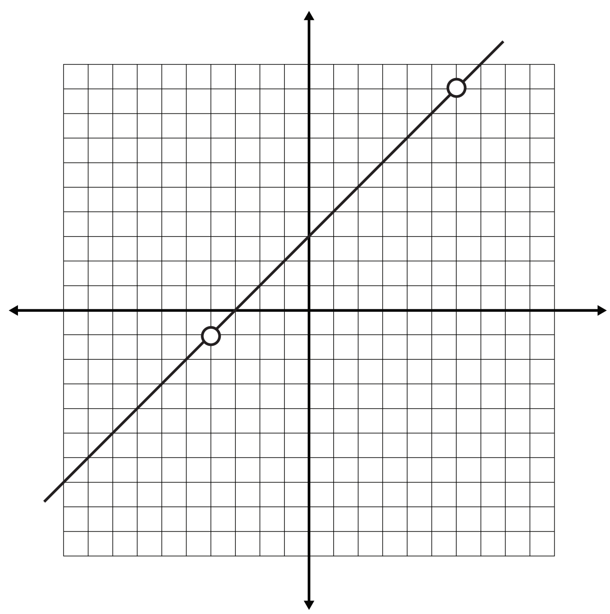
b)



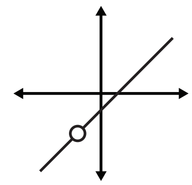
c)



d)



Lesson Notes



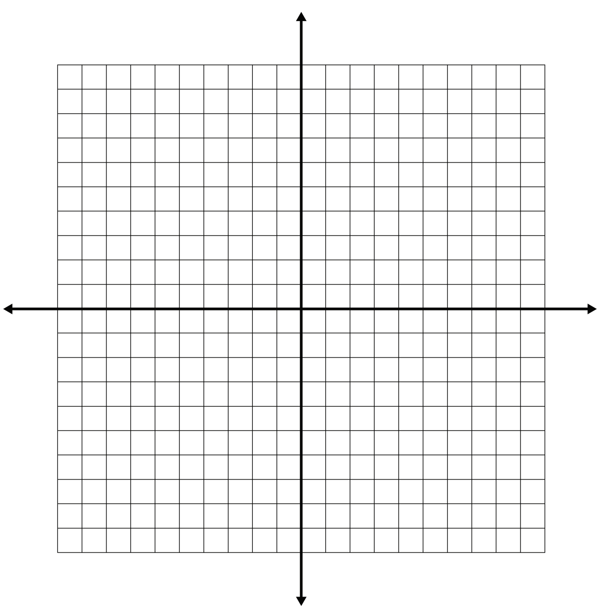
$$y = \frac{x^2 + x - 2}{x + 2}$$

Example 10 Solve the rational equation $\frac{3x}{x - 1} = 4$ in three different ways.

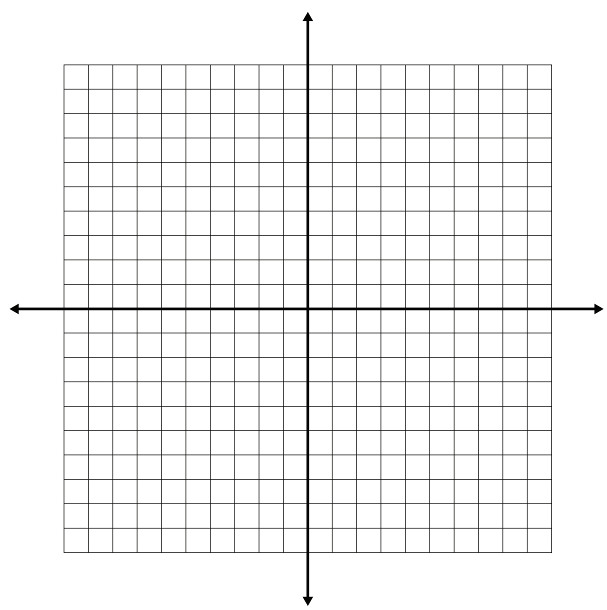
Rational Equations

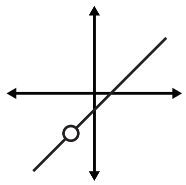
a) Solve algebraically and check for extraneous roots.

b) Solve the equation by finding the point of intersection of a system of functions.



c) Solve the equation by finding the x-intercept(s) of a single function.





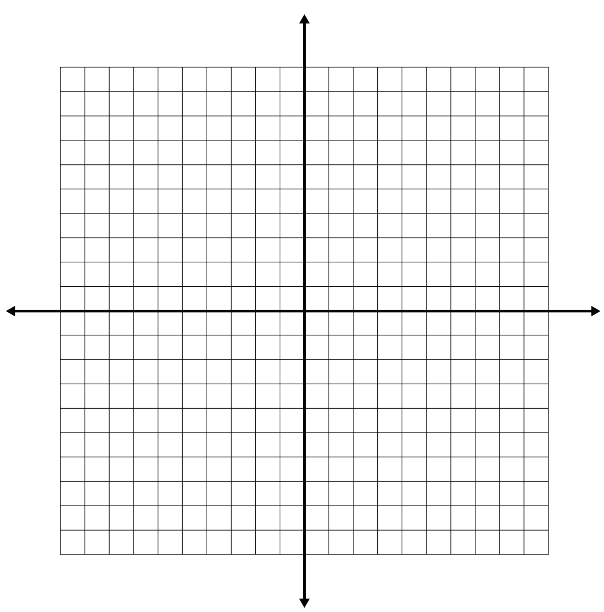
$y = \frac{x^2 + x - 2}{x + 2}$

Example 11 Solve the rational equation $\frac{6}{x} - \frac{9}{x - 1} = -6$ in three different ways.

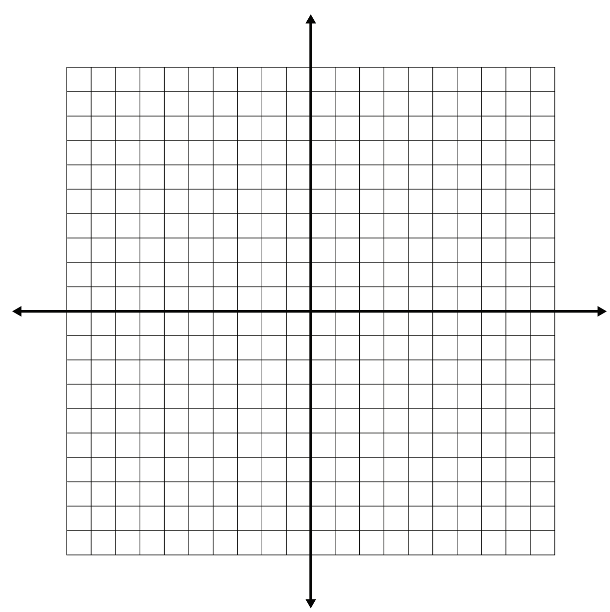
Rational Equations

a) Solve algebraically and check for extraneous roots.

b) Solve the equation by finding the point of intersection of a system of functions.



c) Solve the equation by finding the x-intercept(s) of a single function.



Polynomial, Radical, and Rational Functions
LESSON SIX - *Rational Functions II*
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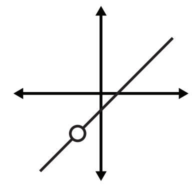


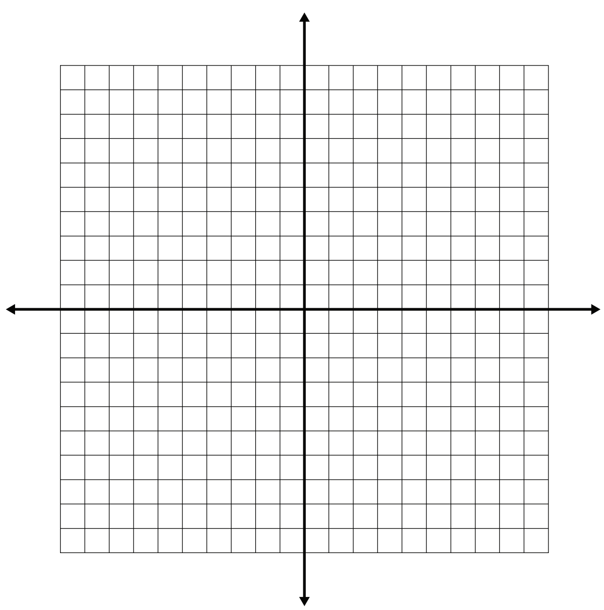
$$y = \frac{x^2 + x - 2}{x + 2}$$

Example 12 Solve the equation $\dfrac{x}{x - 2} - \dfrac{4}{x + 1} = \dfrac{6}{x^2 - x - 2}$ in three different ways.

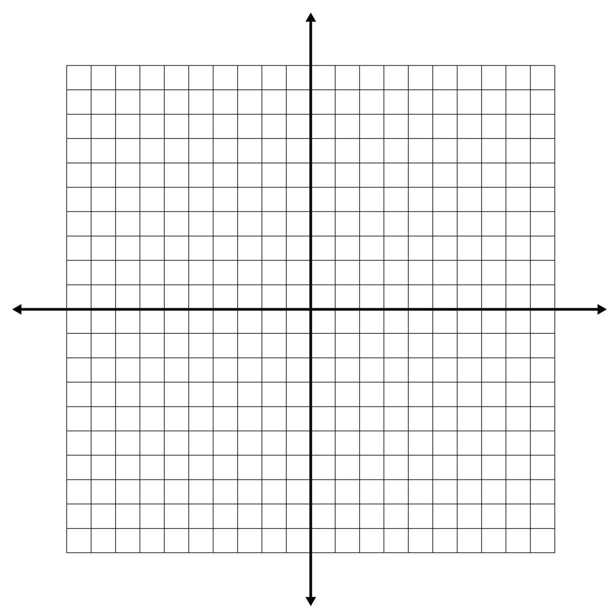
Rational Equations

a) Solve algebraically and check for extraneous roots.

b) Solve the equation by finding the point of intersection of a system of functions.



c) Solve the equation by finding the x-intercept(s) of a single function.



Example 13

Cynthia jogs 3 km/h faster than Alan. In a race, Cynthia was able to jog 15 km in the same time it took Alan to jog 10 km. How fast were Cynthia and Alan jogging?

a) Fill in the table and derive an equation that can be used to solve this problem.

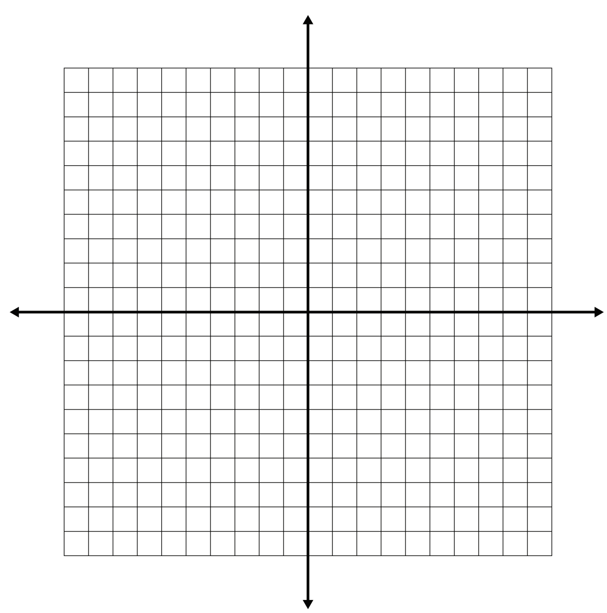
	d	*s*	*t*
Cynthia			
Alan			

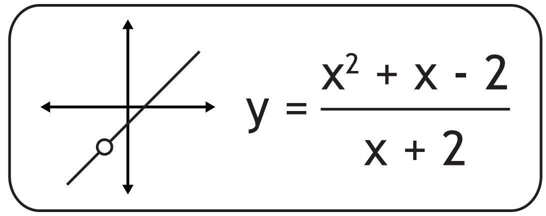
b) Solve algebraically.

c) Check your answer by either:
i) finding the point of intersection of two functions.

OR

ii) finding the x-intercept(s) of a single function.





Example 14

George can canoe 24 km downstream and return to his starting position (upstream) in 5 h. The speed of the current is 2 km/h. What is the speed of the canoe in still water?

a) Fill in the table and derive an equation that can be used to solve this problem.

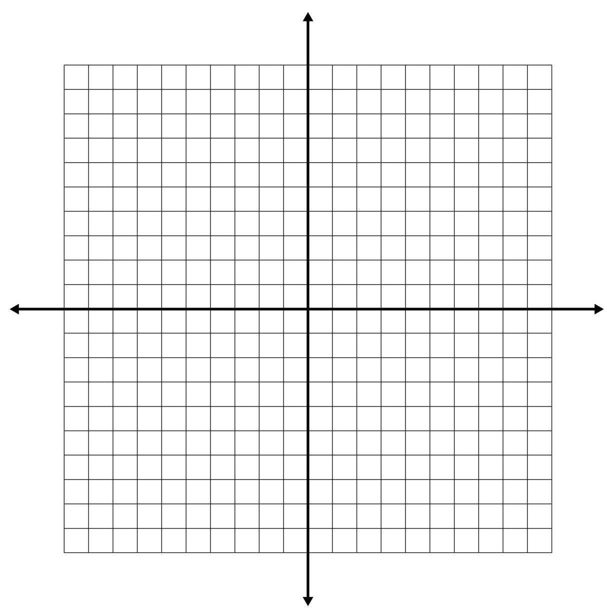
	d	*s*	*t*
Upstream			
Downstream			

b) Solve algebraically.

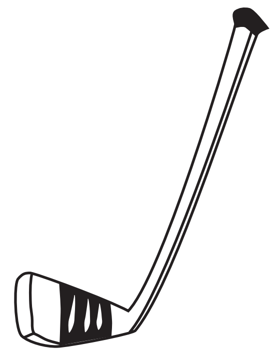
c) Check your answer by either:
i) finding the point of intersection of two functions.

OR

ii) finding the x-intercept(s) of a single function.



Example 15



The shooting percentage of a hockey player is ratio of scored goals to total shots on goal. So far this season, Laura has scored 2 goals out of 14 shots taken. Assuming Laura scores a goal with every shot from now on, how many goals will she need to have a 40% shooting percentage?

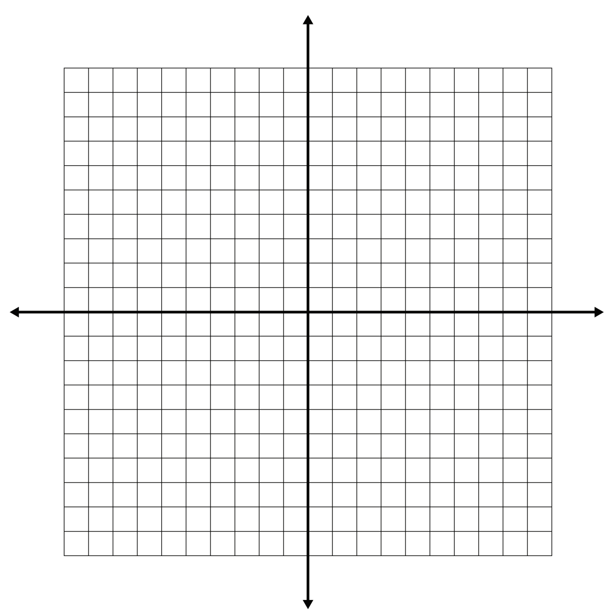
a) Derive an equation that can be used to solve this problem.

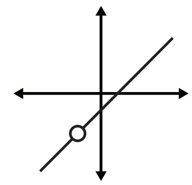
b) Solve algebraically.

c) Check your answer by either:
i) finding the point of intersection of two functions.

OR

ii) finding the x-intercept(s) of a single function.





$$y = \frac{x^2 + x - 2}{x + 2}$$

Example 16

A 300 g mixture of nuts contains peanuts and almonds. The mixture contains 35% almonds by mass. What mass of almonds must be added to this mixture so it contains 50% almonds?

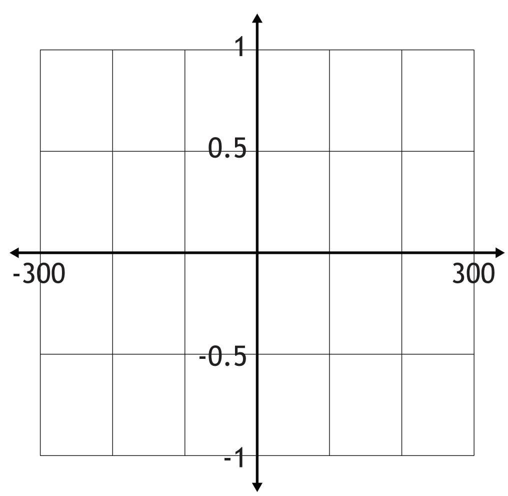
a) Derive an equation that can be used to solve this problem.

b) Solve algebraically.

c) Check your answer by either:
i) finding the point of intersection of two functions.

OR

ii) finding the x-intercept(s) of a single function.



Answer Key

Polynomial, Radical, and Rational Functions Lesson One: Polynomial Functions

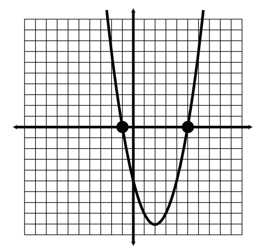
Example 1: a) Leading coefficient is a_n ; polynomial degree is n; constant term is a_0. i) 3; 1; -2 ii) 1; 3; -1 iii) 5; 0; 5

b) i) Y ii) N iii) Y iv) N v) Y vi) N vii) N viii) Y ix) N

Example 2: a) i) Even-degree polynomials with a positive leading coefficient have a trendline that matches an upright parabola.
End behaviour: The graph starts in the upper-left quadrant (II) and ends in the upper-right quadrant (I).
ii) Even-degree polynomials with a negative leading coefficient have a trendline that matches an upside-down parabola.
End behaviour: The graph starts in the lower-left quadrant (III) and ends in the lower-right quadrant (IV).

b) i) Odd-degree polynomials with a positive leading coefficient have a trendline matching the line y = x.
The end behaviour is that the graph starts in the lower-left quadrant (III) and ends in the upper-right quadrant (I).
ii) Odd-degree polynomials with a negative leading coefficient have a trendline matching the line y = -x.
The end behaviour is that the graph starts in the upper-left quadrant (II) and ends in the lower-right quadrant (IV).

Example 3: a) Zero of a Polynomial Function: Any value of x that satisfies the equation P(x) = 0 is called a zero of the polynomial. A polynomial can have several unique zeros, duplicate zeros, or no real zeros. i) Yes; P(-1) = 0 ii) No; P(3) ≠ 0.
b) Zeros: -1, 5.
c) The x-intercepts of the polynomial's graph are -1 and 5.
These are the same as the zeros of the polynomial.
d) "Zero" describes a property of a function; "Root" describes a property of an equation; and "x-intercept" describes a property of a graph.

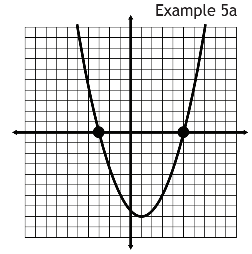


Example 4: a) Multiplicity of a Zero: The multiplicity of a zero (or root) is how many times the root appears as a solution.
Zeros give an indication as to how the graph will behave near the x-intercept corresponding to the root.
b) Zeros: -3 (multiplicity 1) and 1 (multiplicity 1). **c)** Zero: 3 (multiplicity 2).
d) Zero: 1 (multiplicity 3). **e)** Zeros: -1 (multiplicity 2) and 2 (multiplicity 1).

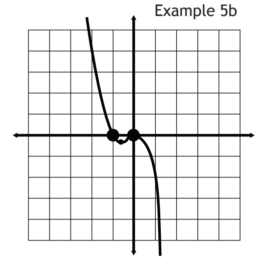
Example 5: a) i) Zeros: -3 (multiplicity 1) and 5 (multiplicity 1).
ii) y-intercept: (0, -7.5). iii) End behaviour: graph starts in QII, ends in QI.
iv) Other points: parabola vertex (1, -8).

b) i) Zeros: -1 (multiplicity 1) and 0 (multiplicity 2). ii) y-intercept: (0, 0).
iii) End behaviour: graph starts in QII, ends in QIV.
iv) Other points: (-2, 4), (-0.67, -0.15), (1, -2).

Example 5a



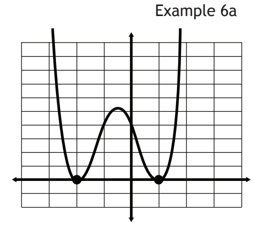
Example 5b



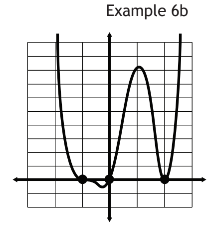
Example 6: a) i) Zeros: -2 (multiplicity 2) and 1 (multiplicity 2).
ii) y-intercept: (0, 4). iii) End behaviour: graph starts in QII, ends in QI.
iv) Other points: (-3, 16), (-0.5, 5.0625), (2, 16).

b) i) Zeros: -1 (multiplicity 3), 0 (multiplicity 1), and 2 (multiplicity 2).
ii) y-intercept: (0, 0). iii) End behaviour: graph starts in QII, ends in QI.
iv) Other points: (-2, 32), (-0.3, -0.5), (1.1, 8.3), (3, 192).

Example 6a



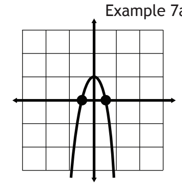
Example 6b



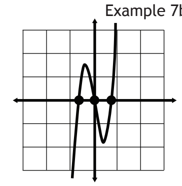
Example 7: a) i) Zeros: -0.5 (multiplicity 1) and 0.5 (multiplicity 1).
ii) y-intercept: (0, 1). iii) End behaviour: graph starts in QIII, ends in QIV.
iv) Other points: parabola vertex (0, 1).

b) i) Zeros: -0.67 (multiplicity 1), 0 (multiplicity 1), and 0.75 (multiplicity 1).
ii) y-intercept: (0, 0). iii) End behaviour: graph starts in QIII, ends in QI.
iv) Other points: (-1, -7), (-0.4, 1.5), (0.4, -1.8), (1, 5).

Example 7a



Example 7b



Example 8:

a) $P(x) = -\frac{1}{3}(x+3)(x-4)$

b) $P(x) = \frac{1}{8}(x+2)^3(x-1)$

Example 9:

a) $P(x) = -\frac{1}{12}x^3(x-5)^2$

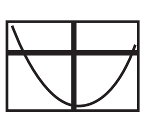
b) $P(x) = \frac{1}{32}(x+6)(x+2)(x-2)(x-6)$

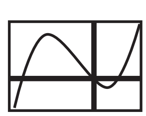
Example 10:

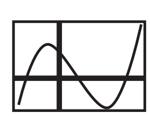
a) $P(x) = \frac{1}{2}(2x+3)(3x-4)$

b) $P(x) = \frac{1}{288}(x+6)^2(3x+8)(4x-9)$

Example 11: **a)** x: [-15, 15, 1], y: [-169, 87, 1] **b)** x: [-12, 7, 1], y: [-192, 378, 1] **c)** x: [-12, 24, 1], y: [-1256, 2304, 1]

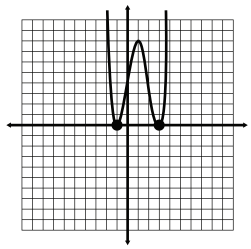




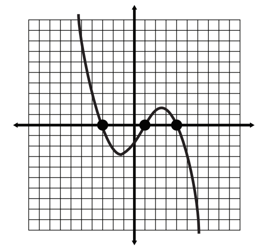


Answer Key

Example 12: a) $P(x) = \frac{1}{2}(x+1)^2(x-3)^2$

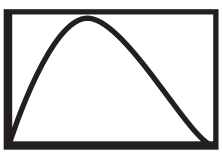


b) $P(x) = -\frac{1}{8}(x+3)(x-1)(x-4)$



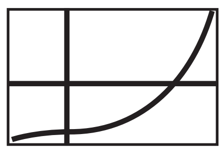
Example 13:

a) V(x) = x(20 - 2x)(16 - 2x)
b) 0 < x < 8 or (0, 8)
c) Window Settings:
x: [0,8, 1], y: [0, 420, 1]
d) When the side length of a corner square is 2.94 cm, the volume of the box will be maximized at 420.11 cm^3.
e) The volume of the box is greater than 200 cm^3 when 0.74 < x < 5.93. or (0.74, 5.93)



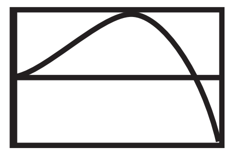
Example 14:

a) $P_{product}(x) = x^2(x + 2)$; $P_{sum}(x) = 3x + 2$
b) $x^3 + 2x^2 - 3x - 11550 = 0$.
c) Window Settings:
x: [-10, 30, 1], y: [-12320, 17160, 1]
Quinn and Ralph are 22 since x = 22.
Audrey is two years older, so she is 24.



Example 15:

a) Window Settings:
x: [0, 6, 1], y: [-1.13, 1.17, 1]
b) At 3.42 seconds, the maximum volume of 1.17 L is inhaled
c) One breath takes 5.34 seconds to complete.
d) 64% of the breath is spent inhaling.



Example 16: $V(h) = \frac{1}{4}\pi(64h - h^3)$

Polynomial, Radical, and Rational Functions Lesson Two: Polynomial Division

Example 1: a) Quotient: $x^2 - 5$; R = 4 **b)** P(x): $x^3 + 2x^2 - 5x - 6$; D(x) = x + 2; Q(x) = $x^2 - 5$; R = 4 **c)** L.S.= R.S. **d)** Q(x) = $x^2 - 5 + 4/(x + 2)$ **e)** Q(x) = $x^2 - 5 + 4/(x + 2)$

Example 2: a) $3x^2 - 7x + 9 - 10/(x + 1)$ **b)** $x^2 + 2x + 1$ **c)** $x^2 - 2x + 4 - 9/(x + 2)$

Example 3: a) $3x^2 + 3x + 2 - 1/(x - 1)$ **b)** $3x^3 - x^2 + 2x - 1$ **c)** $2x^3 + 2x^2 - 5x - 5 - 1/(x - 1)$

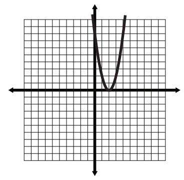
Example 4: a) x - 2 **b)** 2 **c)** $x^2 - 4$ **d)** $x^2 + 5x + 12 + 36/(x - 3)$

Example 5: a) a = -5 **b)** a = -5

Example 6: The dimensions of the base are x + 5 and x - 3

Example 7: a) $f(x) = 2(x + 1)(x - 2)^2$ **b)** g(x) = x + 1 **c)** $Q(x) = 2(x - 2)^2$

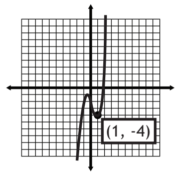
Example 7c



Example 8: a) $f(x) = 4x^3 - 7x - 3$ **b)** g(x) = x - 1

Example 9: a) R = -4
b) R = -4. The point (1, -4) exists on the graph. The remainder is just the y-value of the graph when x = 1.
c) Both synthetic division and the remainder theorem return a result of -4 for the remainder.
d) i) R = 4 ii) R = -2 iii) R = -2
e) When the polynomial P(x) is divided by x - a, the remainder is P(a).

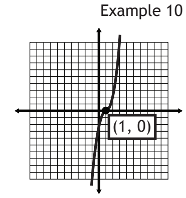
Example 9



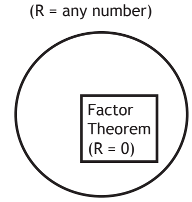
Example 11: a) P(-1) ≠ 0, so x + 1 is not a factor.
b) P(-2) ≠ 0, so x + 2 is not a factor.
c) P(1/3) = 0, so 3x - 1 is a factor.
d) P(-3/2) ≠ 0, so 2x + 3 is not a factor.
Example 12: a) k = 3 **b)** k = -7 **c)** k = -7 **d)** k = -5
Example 13: m = 4 and n = -7
Example 14: m = 4 and n = -3
Example 15: a = 5

Example 10: a) R = 0
b) R = 0. The point (1, 0) exists on the graph. The remainder is just the y-value of the graph.
c) Both synthetic division and the remainder theorem return a result of 0 for the remainder.
d) If P(x) is divided by x - a, and P(a) = 0, then x - a is a factor of P(x).
e) When we use the remainder theorem, the result can be any real number. If we use the remainder theorem and get a result of zero, the factor theorem gives us one additional piece of information - the divisor fits evenly into the polynomial and is therefore a factor of the polynomial. Put simply, we're always using the remainder theorem, but in the special case of R = 0 we get extra information from the factor theorem.

Example 10



Remainder Theorem
(R = any number)



Interval Notation

Math 30-1 students are expected to know that domain and range can be expressed using ***interval notation***.

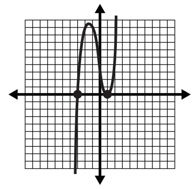
() - Round Brackets: Exclude point from interval.
[] - Square Brackets: Include point in interval.
Infinity ∞ always gets a round bracket.

Examples: x ≥ -5 becomes [-5, ∞);
1 < x ≤ 4 becomes (1, 4];
x ε R becomes (-∞ , ∞);
-8 ≤ x < 2 or 5 ≤ x < 11 becomes [-8, 2) U [5, 11), where U means "or", or ***union of sets***;
x ε R, x ≠ 2 becomes (-∞ , 2) U (2, ∞);
-1 ≤ x ≤ 3, x ≠ 0 becomes [-1 , 0) U (0, 3].

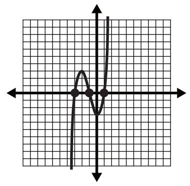
Answer Key

Polynomial, Radical, and Rational Functions Lesson Three: Polynomial Factoring

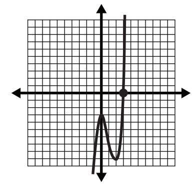
Example 1: a) The integral factors of the constant term of a polynomial are potential zeros of the polynomial.
b) Potential zeros of the polynomial are ±1 and ±3.
c) The zeros of P(x) are -3 and 1 since P(-3) = 0 and P(1) = 0
d) The x-intercepts match the zeros of the polynomial
e) $P(x) = (x + 3)(x - 1)^2$.



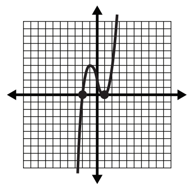
Example 2: a) $P(x) = (x + 3)(x + 1)(x - 1)$.
b) All of the factors can be found using the graph.
c) Factor by grouping.



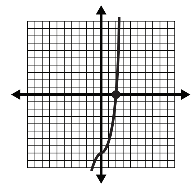
Example 3: a) $P(x) = (2x^2 + 1)(x - 3)$.
b) Not all of the factors can be found using the graph.
c) Factor by grouping.



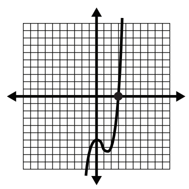
Example 4: a) $P(x) = (x + 2)(x - 1)^2$.
b) All of the factors can be found using the graph.
c) No.



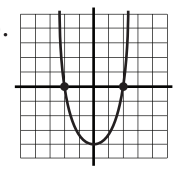
Example 5: a) $P(x) = (x^2 + 2x + 4)(x - 2)$.
b) Not all of the factors can be found using the graph.
c) $x^3 - 8$ is a difference of cubes



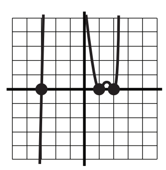
Example 6: a) $P(x) = (x^2 + x + 2)(x - 3)$.
b) Not all of the factors can be found using the graph.
c) No.



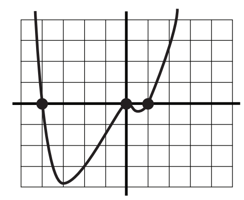
Example 7: a) $P(x) = (x^2 + 4)(x - 2)(x + 2)$.
b) Not all of the factors can be found using the graph.
c) $x^4 - 16$ is a difference of squares.



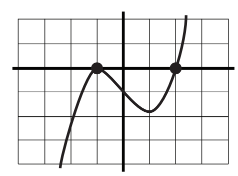
Example 8: a) $P(x) = (x + 3)(x - 1)^2(x - 2)^2$.
b) All of the factors can be found using the graph.
c) No.



Example 9:
a) $P(x) = 1/2x^2(x + 4)(x - 1)$.



b) $P(x) = 2(x + 1)^2(x - 2)$.



Example 10: Width = 10 cm; Height = 7 cm; Length = 15 cm

Example 11: -8; -7; -6

Example 12: k = 2; P(x) = (x + 3)(x - 2)(x - 6)

Example 13: a = -3 and b = -1

Example 14: a) x = -3, 2, and 4 **b)** $x = \frac{-5-\sqrt{37}}{6}, -1, \frac{-5+\sqrt{37}}{6}$

Quadratic Formula

From Math 20-1:
The roots of a quadratic equation with the form $ax^2 + bx + c = 0$ can be found with the ***quadratic formula***:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Answer Key

Polynomial, Radical, and Rational Functions Lesson Four: Radical Functions

Example 1:

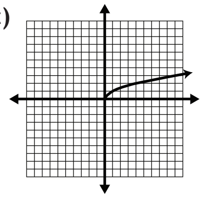
a)

x	$f(x)$
-1	undefined
0	0
1	1
4	2
9	3

b) Domain: $x \geq 0$; Range: $y \geq 0$

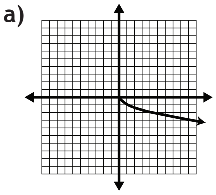
Interval Notation:
Domain: $[0, \infty)$;
Range: $[0, \infty)$

c)

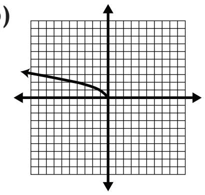


Example 2:

a)

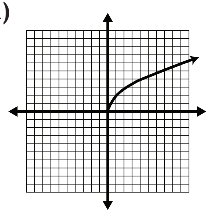


b)

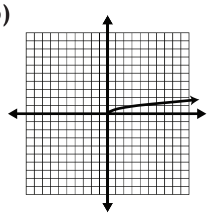


Example 3:

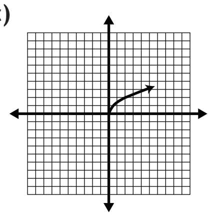
a)



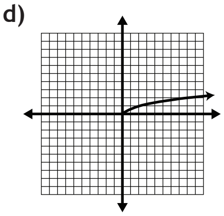
b)



c)

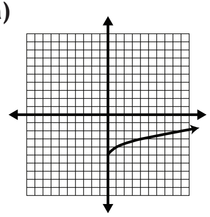


d)

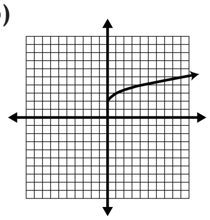


Example 4:

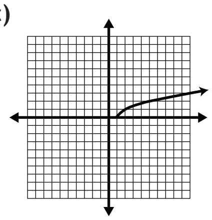
a)



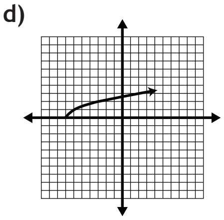
b)



c)

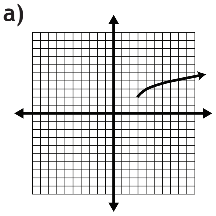


d)

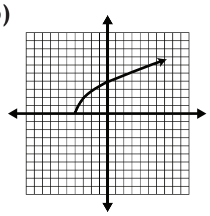


Example 5:

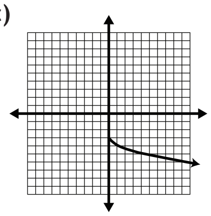
a)



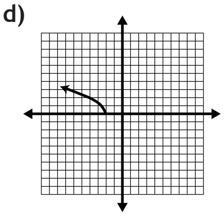
b)



c)

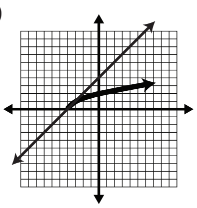


d)



Example 6:

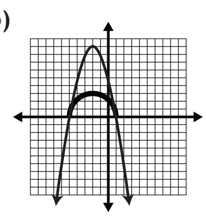
a)



ORIGINAL:
Domain: $x \in R$ or $(-\infty, \infty)$
Range: $y \in R$ or $(-\infty, \infty)$

TRANSFORMED:
Domain: $x \geq -4$ or $[-4, \infty)$
Range: $y \geq 0$ or $[0, \infty)$

b)

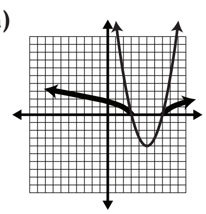


ORIGINAL:
Domain: $x \in R$ or $(-\infty, \infty)$
Range: $y \leq 9$ or $(-\infty, 9]$

TRANSFORMED:
Domain: $-5 \leq x \leq 1$ or $[-5, 1]$
Range: $0 \leq y \leq 3$ or $[0, 3]$

Example 7:

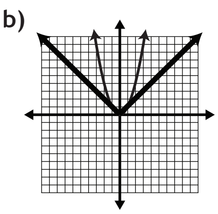
a)



ORIGINAL:
Domain: $x \in R$ or $(-\infty, \infty)$
Range: $y \geq -4$ or $[-4, \infty)$

TRANSFORMED:
Domain: $x \leq 3$ or $x \geq 7$
or $(-\infty, 3] \cup [7, \infty)$
Range: $y \geq 0$ or $[0, \infty)$

b)



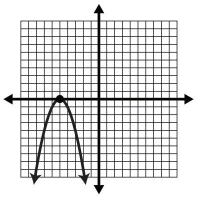
ORIGINAL:
Domain: $x \in R$ or $(-\infty, \infty)$
Range: $y \geq 0$ or $[0, \infty)$

TRANSFORMED:
Domain: $x \in R$ or $(-\infty, \infty)$
Range: $y \geq 0$ or $[0, \infty)$

Answer Key

Example 8:

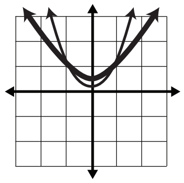
a)



ORIGINAL:
Domain: x ε R or (-∞, ∞)
Range: y ≤ 0 or (-∞, 0]

TRANSFORMED:
Domain: x = -5
Range: y = 0

b)

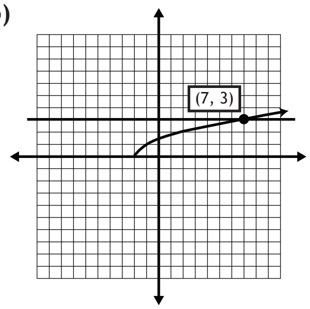


ORIGINAL:
Domain: x ε R or (-∞, ∞)
Range: y ≥ 0.25 or [0.25, ∞)

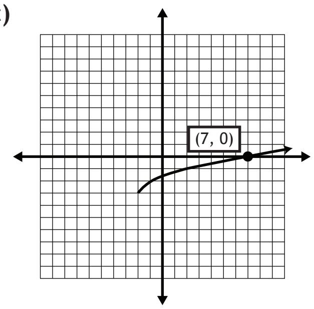
TRANSFORMED:
Domain: x ε R or (-∞, ∞)
Range: y ≥ 0.5 or [0.5, ∞)

Example 9:
a) x = 7

b)

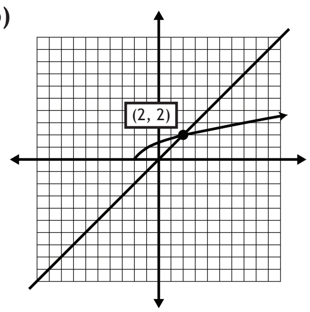


c)

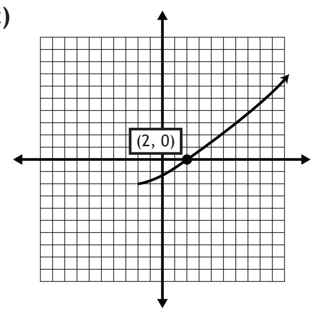


Example 10:
a) x = 2

b)

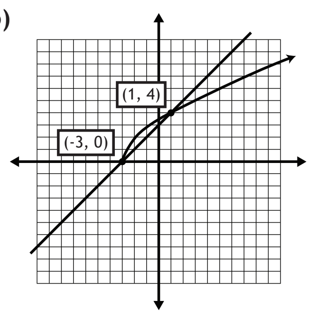


c)

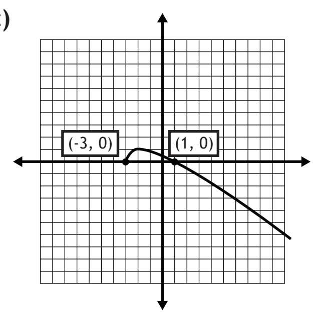


Example 11:
a) x = -3, 1

b)

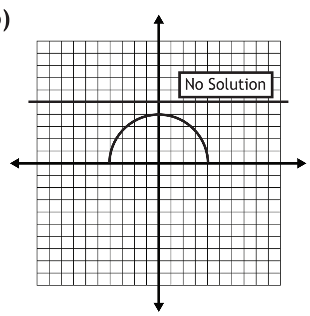


c)

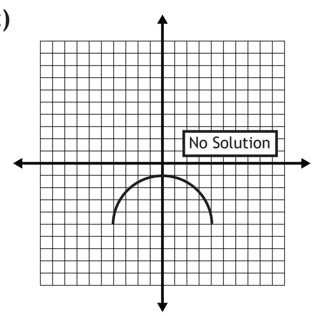


Example 12:
a) No Solution

b)



c)

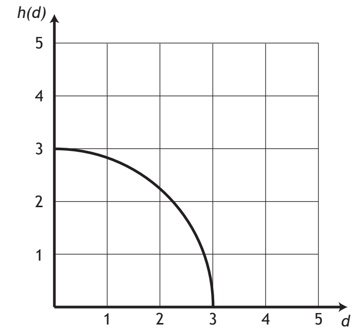


Example 13:

a) $\sqrt{x+2} = 2$ **b)** $\sqrt{x-1} + 2 = -x + 5$ **c)** $-\sqrt{x-4} + 1 = -1$ **d)** $3\sqrt{x} = -3x + 6$

Example 14:

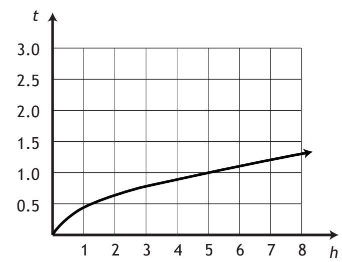
a) $h(d) = \sqrt{9 - d^2}$
b) Domain: 0 ≤ d ≤ 3; Range: 0 ≤ h(d) ≤ 3 or Domain: [0, 3]; Range: [0, 3].
When d = 0, the ladder is vertical.
When d = 3, the ladder is horizontal.
c) 2 m



Example 15:

a) $\sqrt{2}$ × original time
b) 1/2 × original time
c)

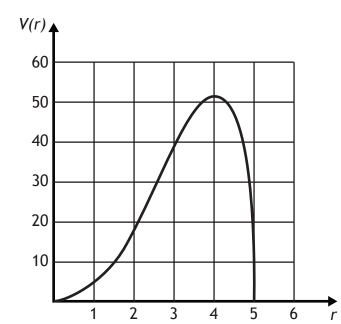
h	t
1	0.4517
4	0.9035
8	1.2778



Example 16:

a) $V(r) = \frac{1}{3}\pi r^2 \sqrt{25 - r^2}$

b)



Answer Key

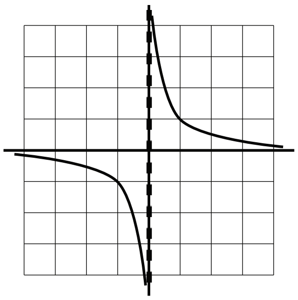
Polynomial, Radical, and Rational Functions Lesson Five: Rational Functions I

Example 1:

a)

x	y
-2	-0.5
-1	-1
-0.5	-2
-0.25	-4
0	undef.
0.25	4
0.5	2
1	1
2	0.5

b)



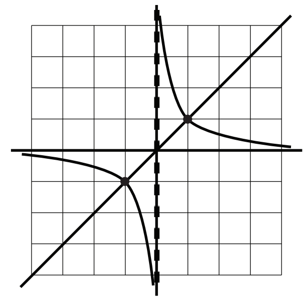
c)

1. The vertical asymptote of the reciprocal graph occurs at the x-intercept of y = x.

2. The invariant points *(points that are identical on both graphs)* occur when y = ±1.

3. When the graph of y = x is below the x-axis, so is the reciprocal graph. When the graph of y = x is above the x-axis, so is the reciprocal graph.

d)

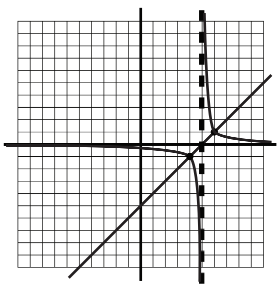


Example 2:

a) Original Graph:
Domain: x ε R or (-∞, ∞);
Range: y ε R or (-∞, ∞)

Reciprocal Graph:
Domain: x ε R, x ≠ 5 or (-∞, 5) U (5, ∞);
Range: y ε R, y ≠ 0 or (-∞, 0) U (0, ∞)

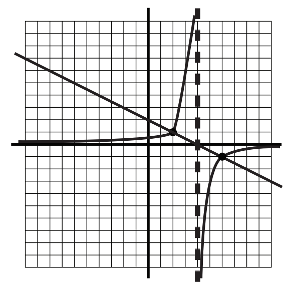
Asymptote Equation(s):
Vertical: x = 5;
Horizontal: y = 0



b) Original Graph:
Domain: x ε R or (-∞, ∞);
Range: y ε R or (-∞, ∞)

Reciprocal Graph:
Domain: x ε R, x ≠ 4 or (-∞, 4) U (4, ∞);
Range: y ε R, y ≠ 0 or (-∞, 0) U (0, ∞)

Asymptote Equation(s):
Vertical: x = 4;
Horizontal: y = 0



Example 3:

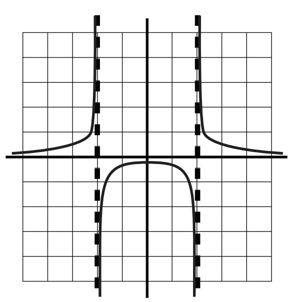
a)

x	y
-3	0.20
-2	undef.
-1	-0.33
0	-0.25
1	-0.33
2	undef.
3	0.20

x	y
-2.05	4.94
-1.95	-5.06

x	y
1.95	-5.06
2.05	4.94

b)



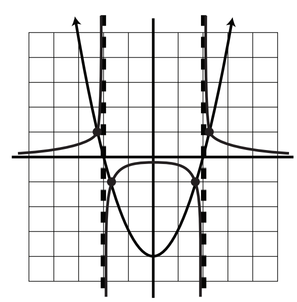
c)

1. The vertical asymptotes of the reciprocal graph occur at the x-intercepts of $y = x^2 - 4$.

2. The invariant points (points that are identical in both graphs) occur when y = ±1.

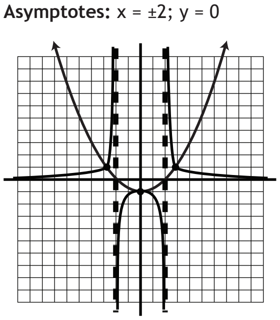
3. When the graph of $y = x^2 - 4$ is below the x-axis, so is the reciprocal graph.
When the graph of $y = x^2 - 4$ is above the x-axis, so is the reciprocal graph.

d)

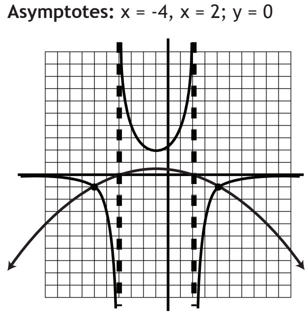


Example 4:

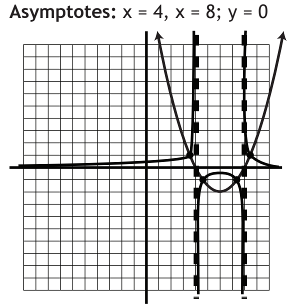
a) Original: x ε R; y ≥ -1
or D: (-∞, ∞); R: [-1, ∞).
Reciprocal: x ε R, x ≠ -2, 2; y ≤ -1 or y > 0
or D: (-∞, -2) U (-2, 2) U (2, ∞); R: (-∞, -1] U (0, ∞)
Asymptotes: x = ±2; y = 0



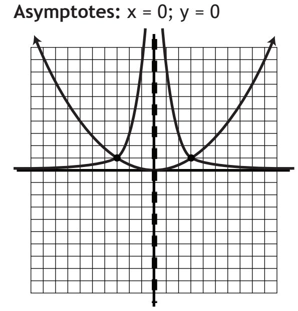
b) Original: x ε R; y ≤ 1/2
or D: (-∞, ∞); R: (-∞, 1/2].
Reciprocal: x ε R, x ≠ -4, 2; y < 0 or y ≥ 2
or D: (-∞, -4) U (-4, 2) U (2, ∞); R: (-∞, 0) U [2, ∞)
Asymptotes: x = -4, x = 2; y = 0



c) Original: x ε R; y ≥ -2
or D: (-∞, ∞); R: [-2, ∞).
Reciprocal: x ε R, x ≠ 4, 8; y ≤ -1/2 or y > 0
or D: (-∞, 4) U (4, 8) U (8, ∞); R: (-∞, -1/2] U (0, ∞)
Asymptotes: x = 4, x = 8; y = 0

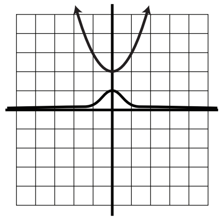


d) Original: x ε R; y ≥ 0
or D: (-∞, ∞); R: [0, ∞).
Reciprocal: x ε R, x ≠ 0; y > 0
or D: (-∞, 0) U (0, ∞); R: (0, ∞)
Asymptotes: x = 0; y = 0

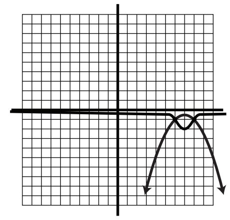


Example 4 (continued):

e) Original: x ε R; y ≥ 2
or D: (-∞, ∞); R: [2, ∞).
Reciprocal: x ε R; 0 < y ≤ 1/2
or D: (-∞, ∞); R: (0, 1/2]
Asymptotes: y = 0

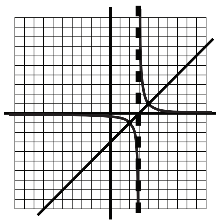


f) Original: x ε R; y ≤ -1/2
or D: (-∞, ∞); R: (-∞, -1/2].
Reciprocal: x ε R; -2 ≤ y < 0
or D: (-∞, ∞); R: [-2, 0)
Asymptotes: y = 0

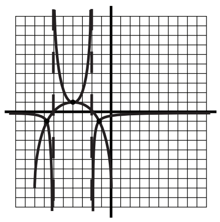


Example 5:

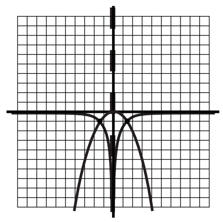
a)



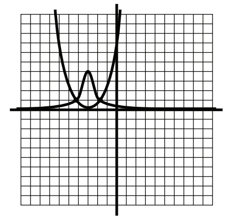
b)



c)

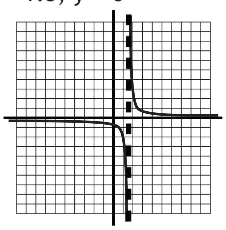


d)

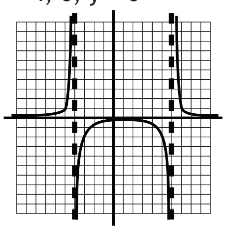


Example 6:

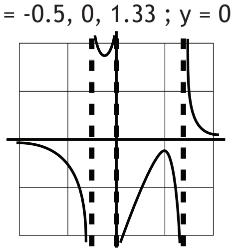
a) x = 1.5; y = 0



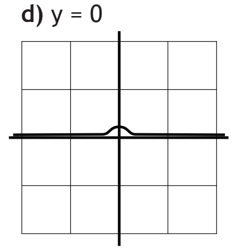
b) x = -4, 6; y = 0



c) x = -0.5, 0, 1.33 ; y = 0

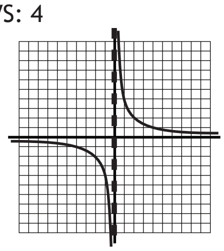


d) y = 0

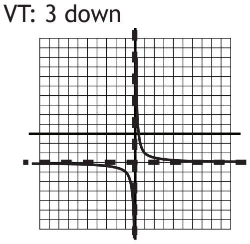


Example 7:

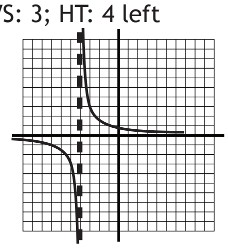
a) VS: 4



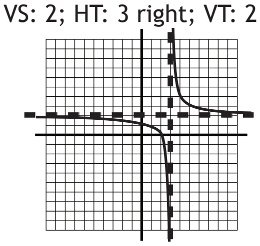
b) VT: 3 down



c) VS: 3; HT: 4 left

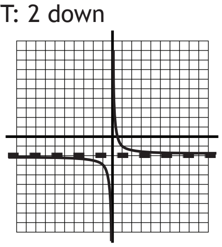


d) VS: 2; HT: 3 right; VT: 2 up

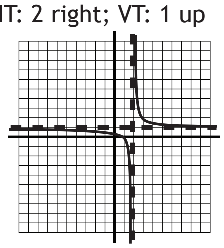


Example 8:

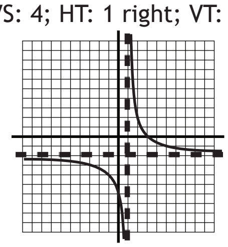
a) VT: 2 down



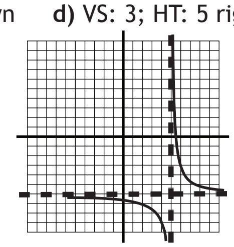
b) HT: 2 right; VT: 1 up



c) VS: 4; HT: 1 right; VT: 2 down



d) VS: 3; HT: 5 right; VT: 2 down



Example 9:

a) P(V) = nRT(1/V).

b) 1/2 × original

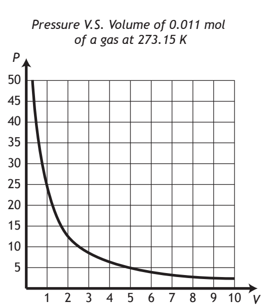
c) 2 × original

d) 8.3 kPa•L/mol•K

e) See table & graph

f) See table & graph

V (L)	P (kPa)
0.5	50
1.0	25
2.0	12.5
5.0	5.0
10.0	2.5



Example 10:

a) 1/4 × original

b) 1/9 × original

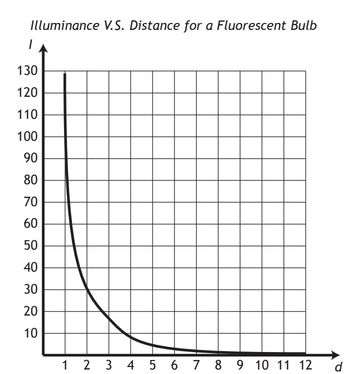
c) 4 × original

d) 16 × original

e) See table & graph

f) See table & graph

d (m)	I (W/m²)
1	$\frac{400}{\pi}$
2	$\frac{100}{\pi}$
4 ORIGINAL	$\frac{25}{\pi}$
8	$\frac{25}{4\pi}$
12	$\frac{25}{9\pi}$

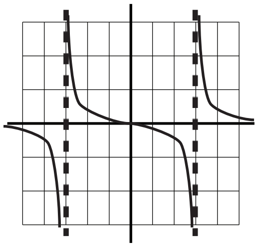


Answer Key

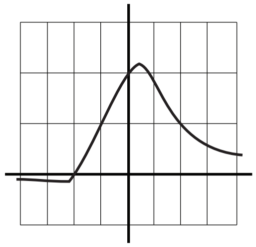
Polynomial, Radical, and Rational Functions Lesson Six: Rational Functions II

Example 1:

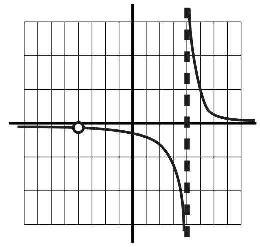
a) $y = \dfrac{x}{x^2 - 9}$



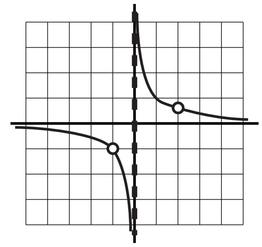
b) $y = \dfrac{x + 2}{x^2 + 1}$



c) $y = \dfrac{x + 4}{x^2 - 16}$

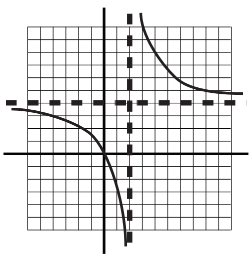


d) $y = \dfrac{x^2 - x - 2}{x^3 - x^2 - 2x}$

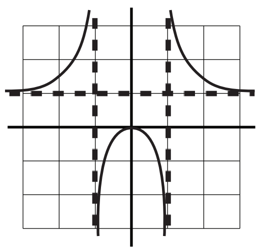


Example 2:

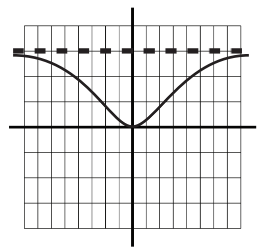
a) $y = \dfrac{4x}{x - 2}$



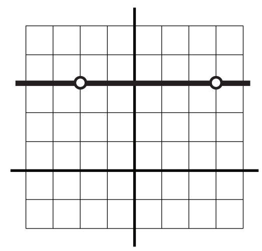
b) $y = \dfrac{x^2}{x^2 - 1}$



c) $y = \dfrac{3x^2}{x^2 + 9}$

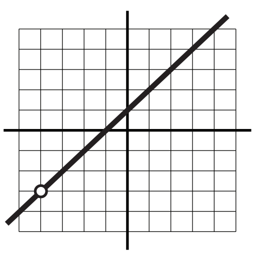


d) $y = \dfrac{3x^2 - 3x - 18}{x^2 - x - 6}$

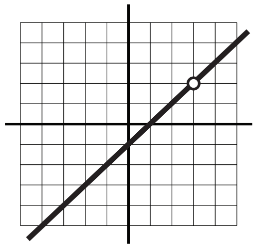


Example 3:

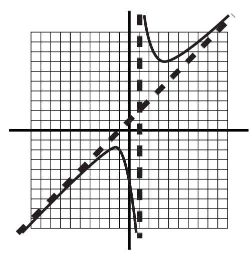
a) $y = \dfrac{x^2 + 5x + 4}{x + 4}$



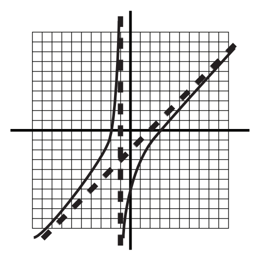
b) $y = \dfrac{x^2 - 4x + 3}{x - 3}$



c) $y = \dfrac{x^2 + 5}{x - 1}$



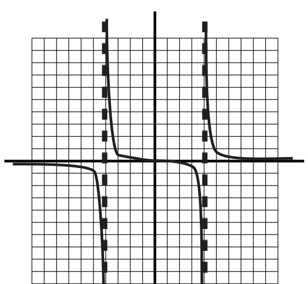
d) $y = \dfrac{x^2 - x - 6}{x + 1}$



Example 4:

i) Horizontal Asymptote: $y = 0$
ii) Vertical Asymptote(s): $x = \pm 4$
iii) y - intercept: (0, 0)
iv) x - intercept(s): (0, 0)
v) Domain: $x \in R, x \neq \pm 4$;
Range: $y \in R$

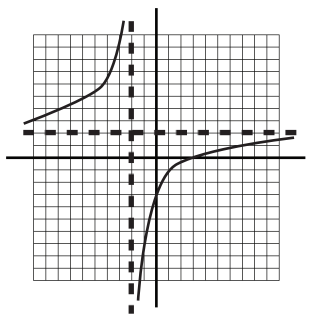
or D: $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$; R: $(-\infty, \infty)$



Example 5:

i) Horizontal Asymptote: $y = 2$
ii) Vertical Asymptote(s): $x = -2$
iii) y - intercept: (0, -3)
iv) x - intercept(s): (3, 0)
v) Domain: $x \in R, x \neq -2$;
Range: $y \in R, y \neq 2$

or D: $(-\infty, -2) \cup (-2, \infty)$; R: $(-\infty, 2) \cup (2, \infty)$

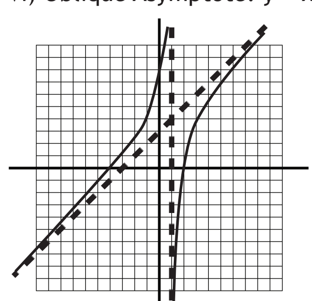


Example 6:

i) Horizontal Asymptote: None
ii) Vertical Asymptote(s): $x = 1$
iii) y - intercept: (0, 8)
iv) x - intercept(s): (-4, 0), (2, 0)
v) Domain: $x \in R, x \neq 1$;
Range: $y \in R$

or D: $(-\infty, 1) \cup (1, \infty)$; R: $(-\infty, \infty)$

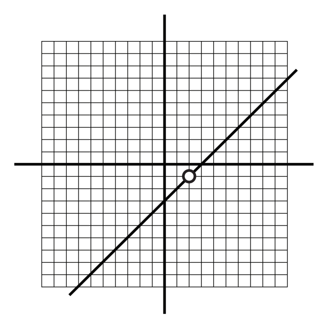
vi) Oblique Asymptote: $y = x + 3$



Example 7:

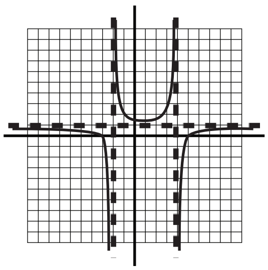
i) $y = x - 3$
ii) Hole: (2, -1)
iii) y - intercept: (0, -3)
iv) x - intercept(s): (3, 0)
v) Domain: $x \in R, x \neq 2$;
Range: $y \in R, y \neq -1$

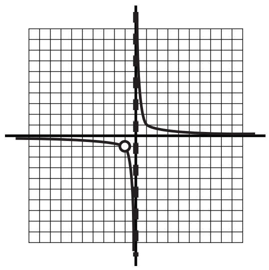
or D: $(-\infty, 2) \cup (2, \infty)$; R: $(-\infty, -1) \cup (-1, \infty)$



Example 8:

a) $y = \dfrac{(x+3)(x-5)}{(x+2)(x-4)}$ b) $y = \dfrac{x+1}{x(x+1)}$





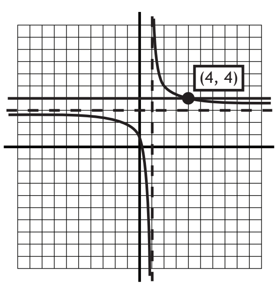
Example 9:

a) $y = \dfrac{x+1}{(x+4)(x-2)}$ b) $y = \dfrac{x(x+3)}{(x+2)(x+3)}$

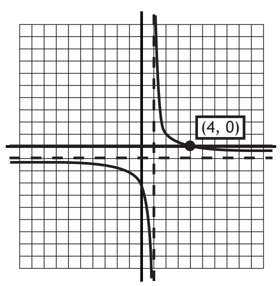
c) $y = \dfrac{7(x+6)(x+2)}{(x+6)(x+2)}$ d) $y = \dfrac{(x+3)(x+4)(x-6)}{(x+4)(x-6)}$

Example 10: **a)** x = 4

b)

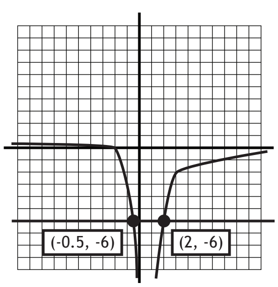


c)

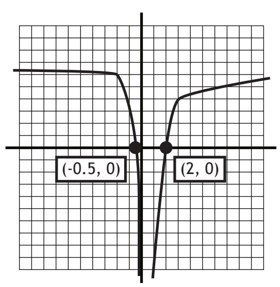


Example 11: **a)** x = -1/2 and x = 2

b)

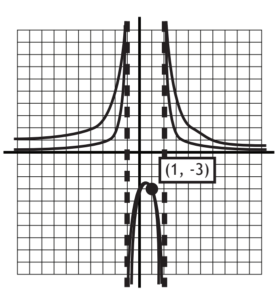


c)

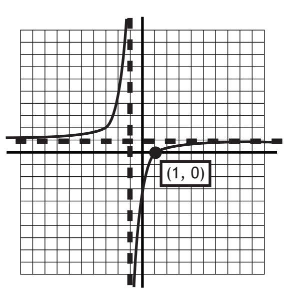


Example 12: **a)** x = 1. x = 2 is an extraneous root

b)



c)



Example 13:

a)

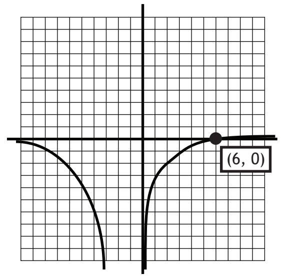
	d	*s*	*t*
Cynthia	15	x + 3	$\frac{15}{x+3}$
Alan	10	x	$\frac{10}{x}$

Equal times.

$$\frac{15}{x+3} = \frac{10}{x}$$

b) Cynthia: 9 km/h; Alan: 6 km/h

c) Graphing Solution: *x-intercept method.*



Example 14:

a)

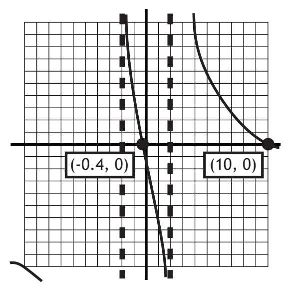
	d	*s*	*t*
Upstream	24	x - 2	$\frac{24}{x-2}$
Downstream	24	x + 2	$\frac{24}{x+2}$

Sum of times equals 5 h.

$$\frac{24}{x-2} + \frac{24}{x+2} = 5$$

b) Canoe speed: 10 km/h

c) Graphing Solution: *x-intercept method.*

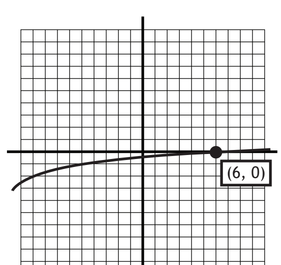


Example 15:

a) $0.40 = \dfrac{2+x}{14+x}$

b) Number of goals required: 6

c) Graphing Solution: *x-intercept method.*



Example 16:

a) $0.50 = \dfrac{105+x}{300+x}$

b) Mass of almonds required: 90 g

c) Graphing Solution: *x-intercept method.*