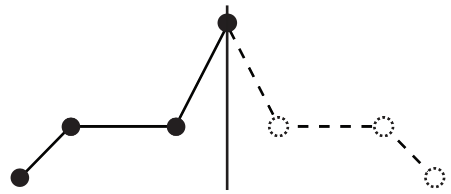
Mathematics 30-1

Student Workbook

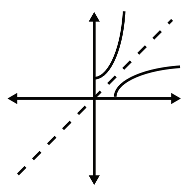
Unit 2



Lesson 1: Basic Transformations
Approximate Completion Time: 2 Days

$$y = af[b(x - h)] + k$$

Lesson 2: Combined Transformations
Approximate Completion Time: 2 Days



$$f^{-1}(x)$$

Lesson 3: Inverses
Approximate Completion Time: 2 Days

$(f + g)(x)$ $(f - g)(x)$

$(f \cdot g)(x)$ $\left(\frac{f}{g}\right)(x)$

Lesson 4: Functions Operations
Approximate Completion Time: 2 Days

$$f \circ g = f(g(x))$$

Lesson 5: Function Composition
Approximate Completion Time: 3 Days

UNIT TWO

Transformations and Operations

Mathematics 30-1

Student Workbook

Unit 2

Complete this workbook by watching the videos on **www.math30.ca**.
Work neatly and use proper mathematical form in your notes.

UNIT TWO

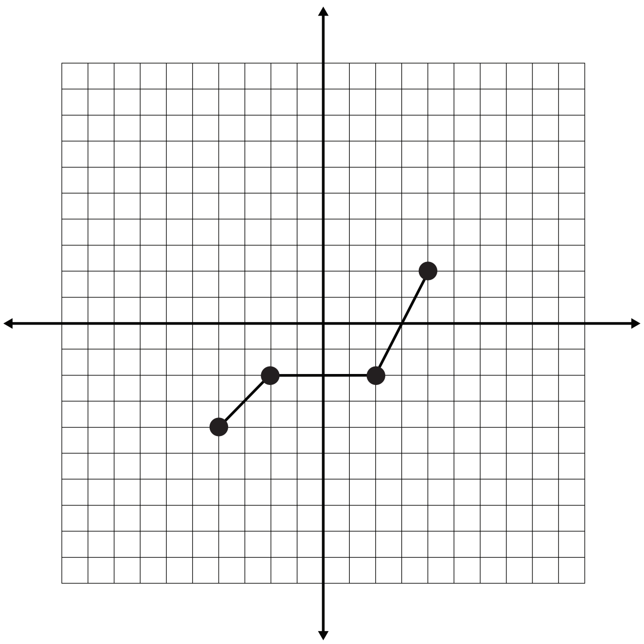
Transformations and Operations

Example 1 Draw the graph resulting from each transformation.
Label the invariant points.

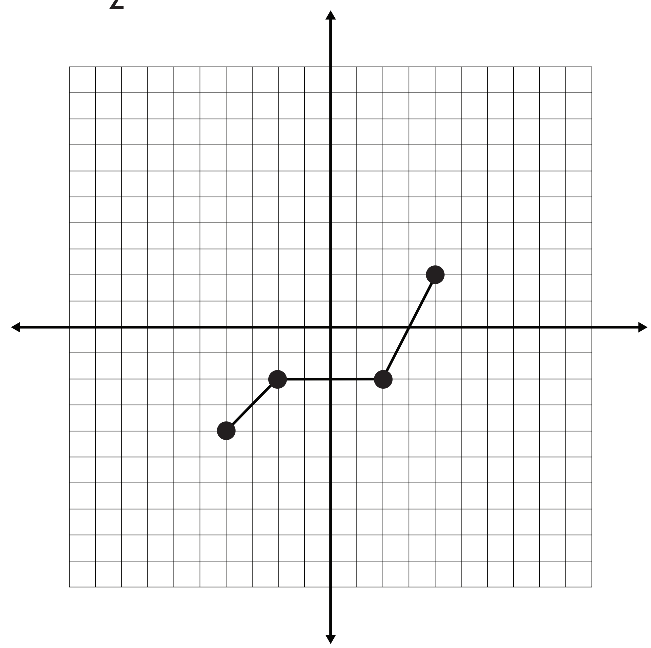
Graphing Stretches

Vertical Stretches

a) $y = 2f(x)$

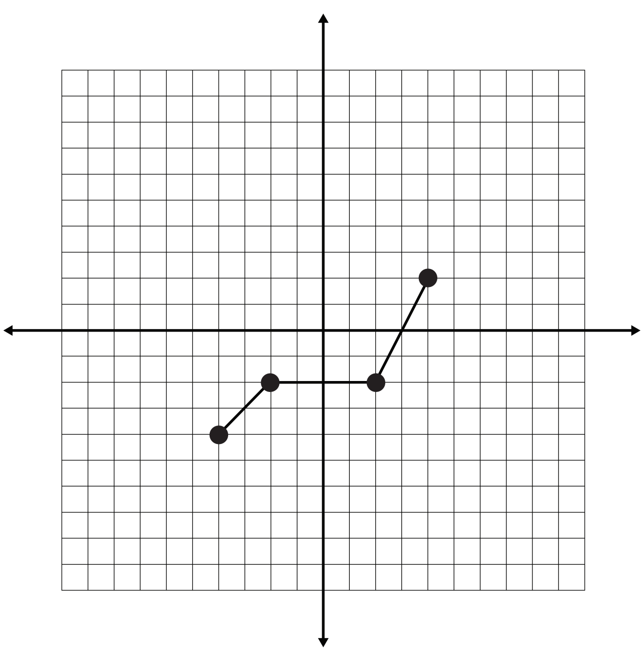


b) $y = \frac{1}{2}f(x)$

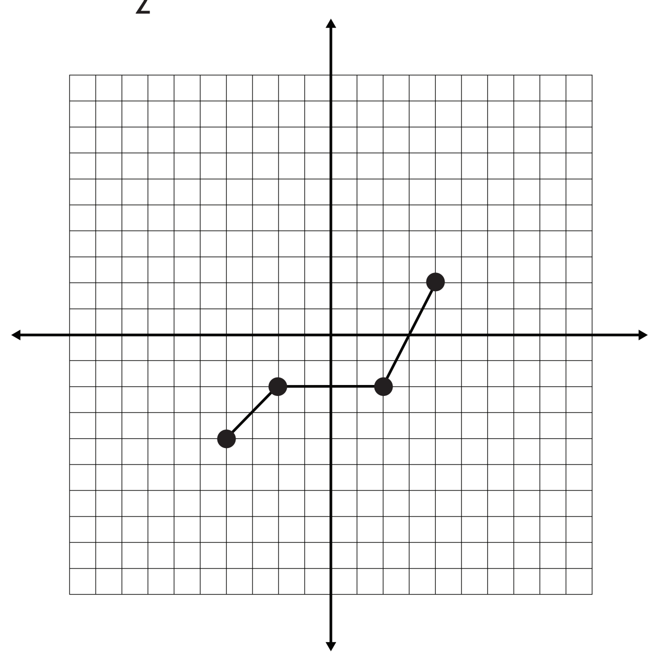


Horizontal Stretches

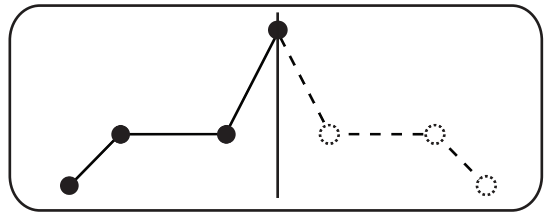
c) $y = f(2x)$



d) $y = f(\frac{1}{2}x)$



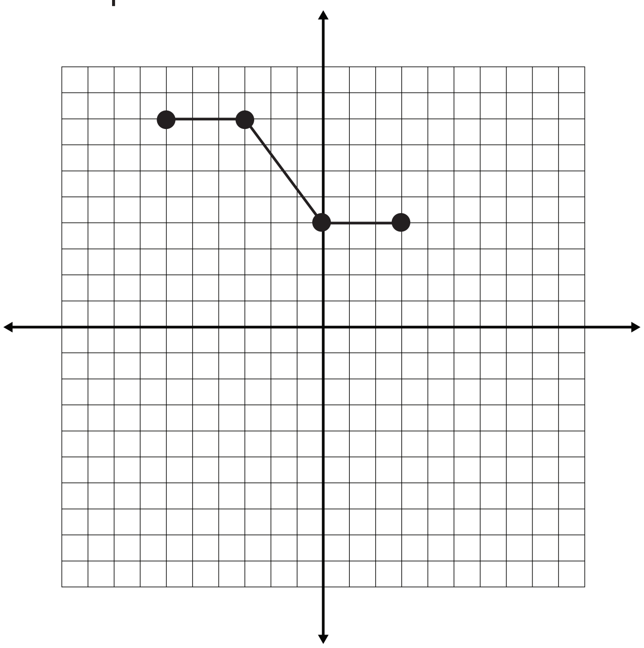
Transformations and Operations
LESSON ONE - *Basic Transformations*
Lesson Notes



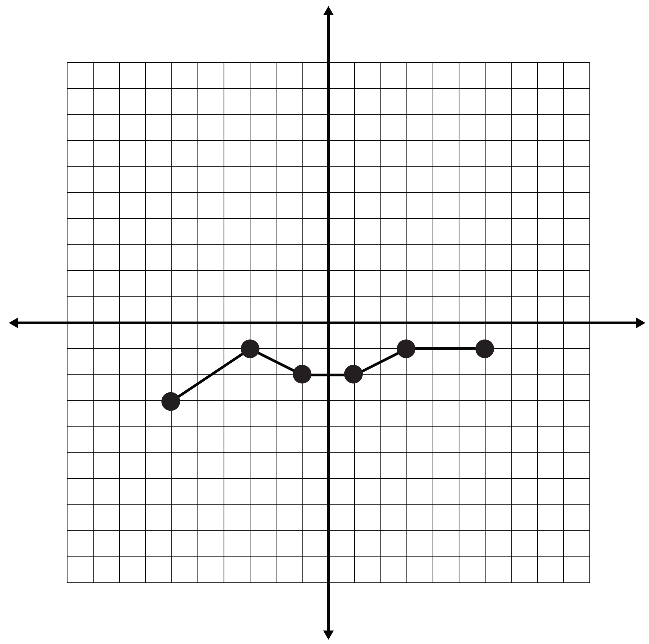
Example 2 Draw the graph resulting from each transformation.
Label the invariant points.

Graphing Stretches

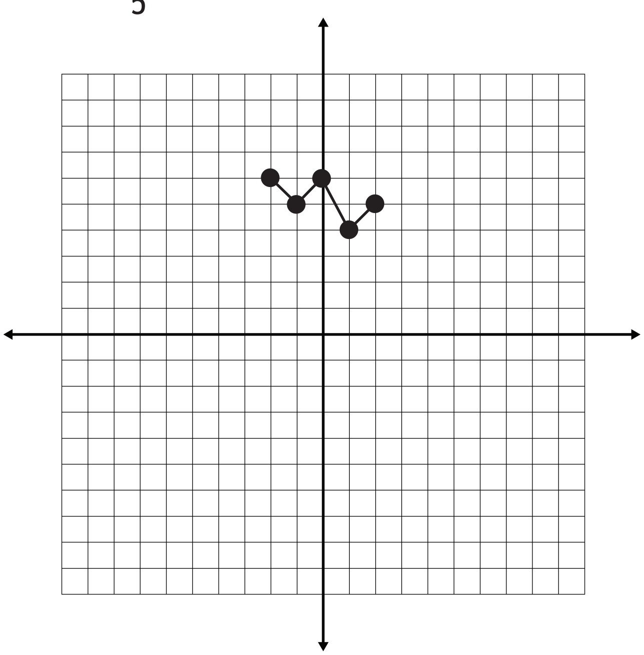
a) $y = \frac{1}{4}f(x)$



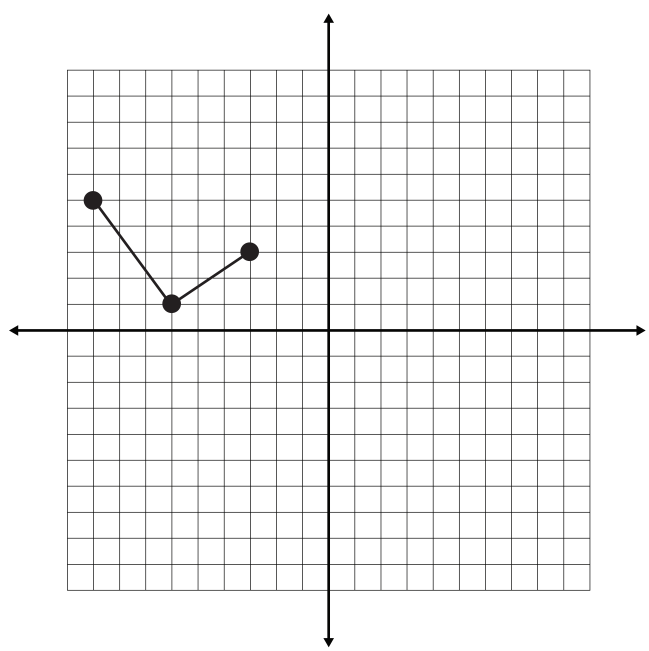
b) $y = 3f(x)$

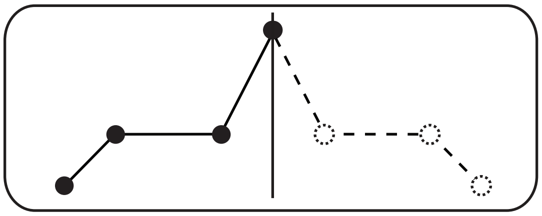


c) $y = f(\frac{1}{5}x)$



d) $y = f(3x)$





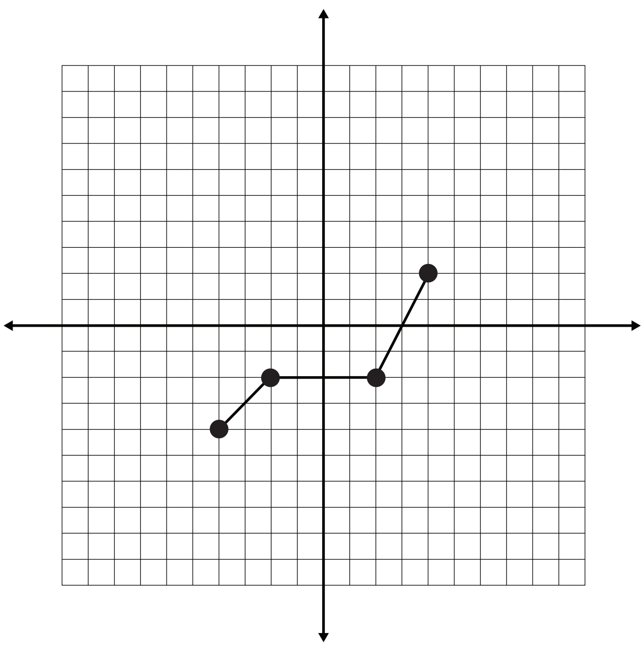
Transformations and Operations
LESSON ONE - *Basic Transformations*
Lesson Notes

Example 3 Draw the graph resulting from each transformation.
Label the invariant points.

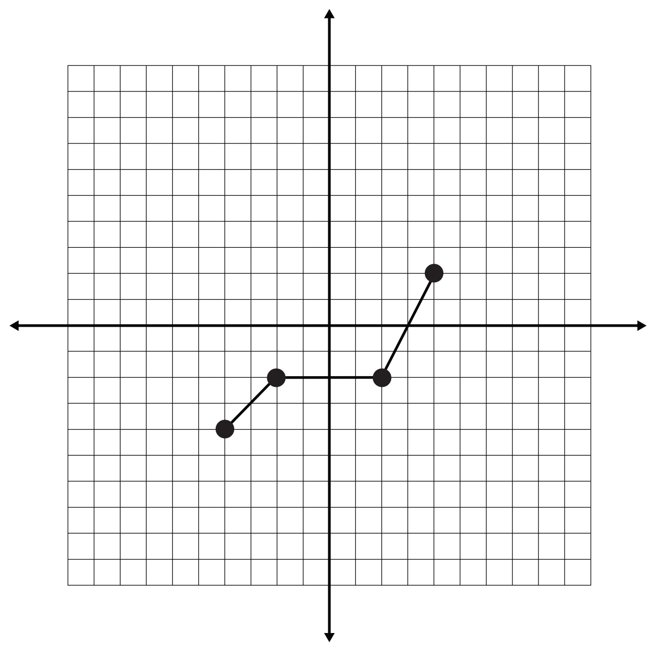
Graphing Reflections

Reflections

a) y = -f(x)

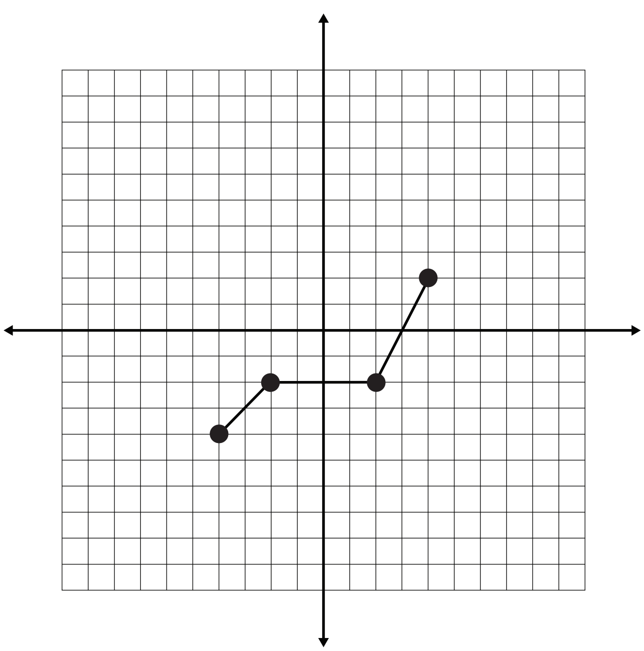


b) y = f(-x)

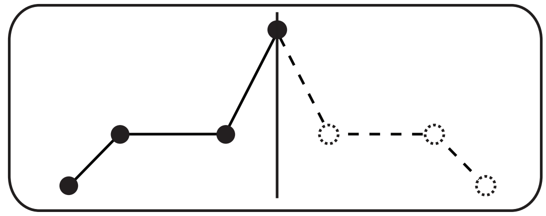


Inverses

c) x = f(y)



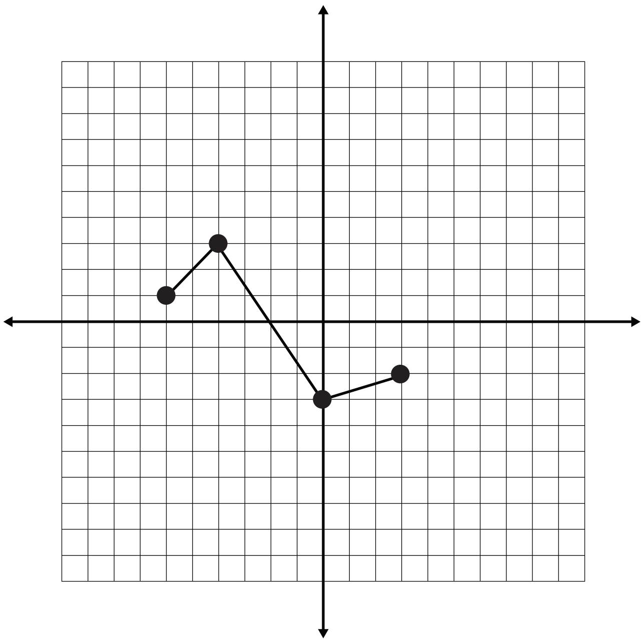
Transformations and Operations
LESSON ONE - *Basic Transformations*
Lesson Notes



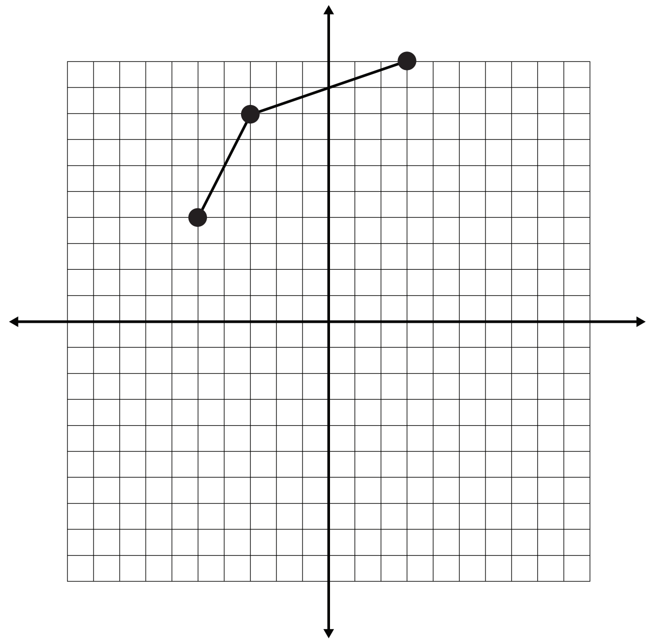
Example 4 Draw the graph resulting from each transformation. Label the invariant points.

Graphing Reflections

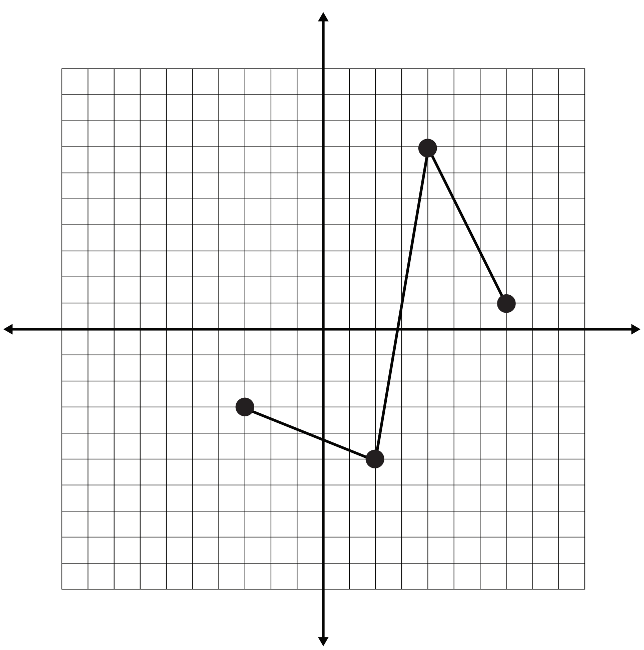
a) $y = -f(x)$

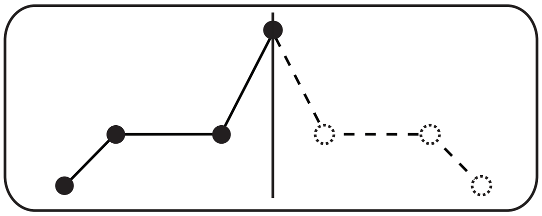


b) $y = f(-x)$



c) $x = f(y)$



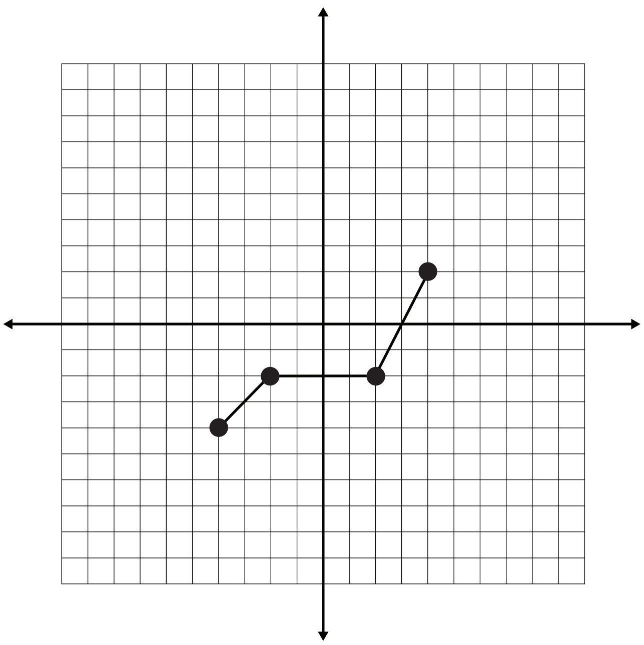


Example 5 Draw the graph resulting from each transformation.

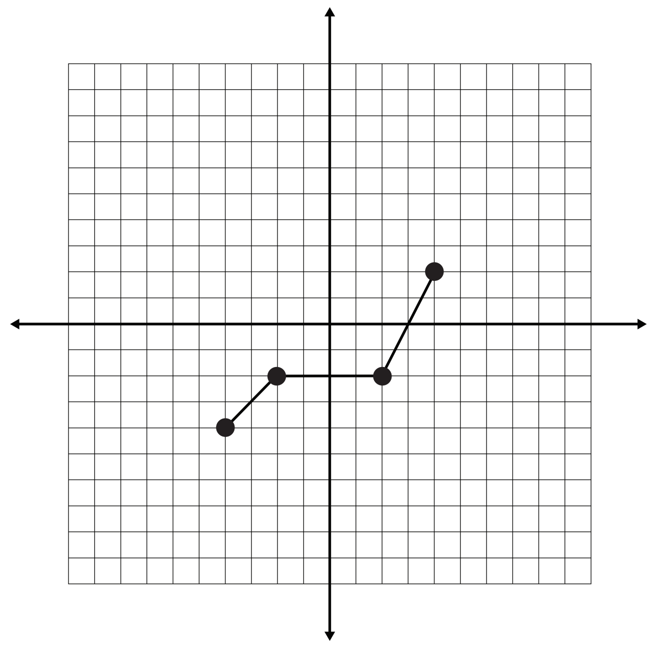
Graphing Translations

Vertical Translations

a) y = f(x) + 3

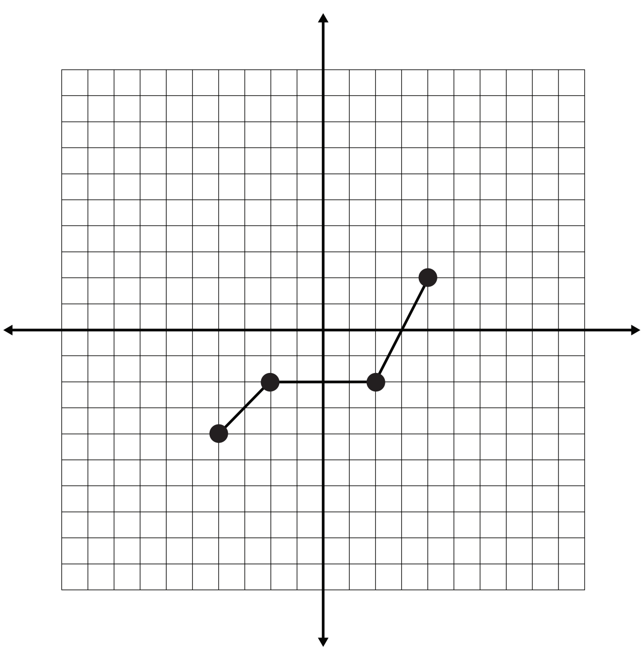


b) y = f(x) - 4

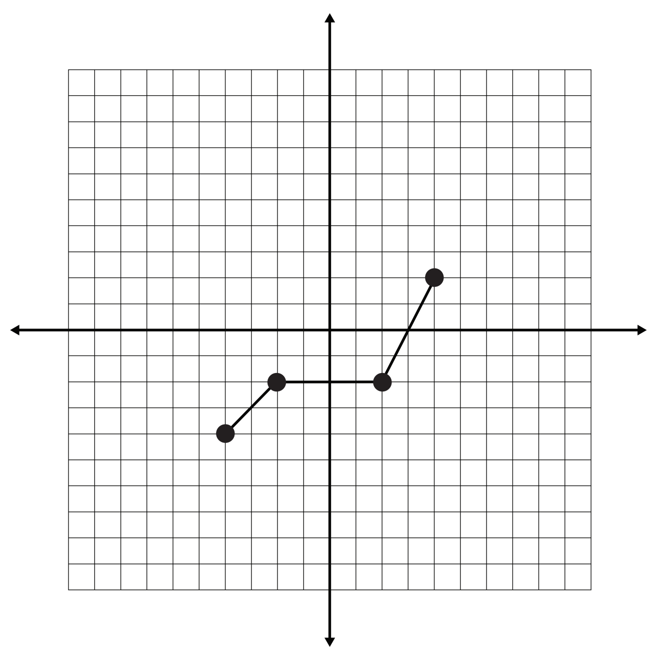


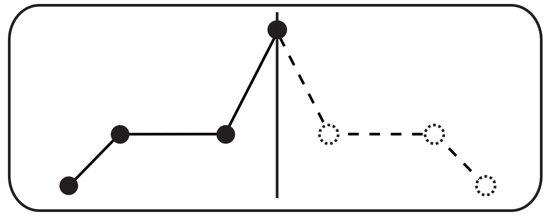
Horizontal Translations

c) y = f(x - 2)



d) y = f(x + 3)

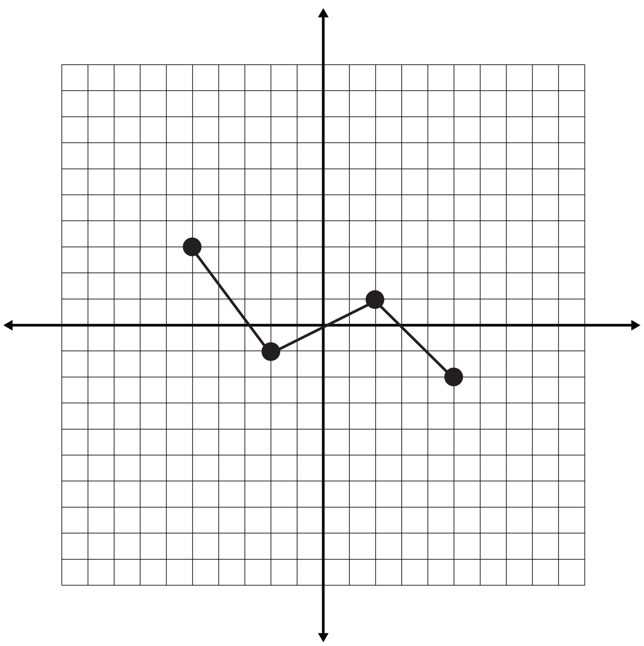




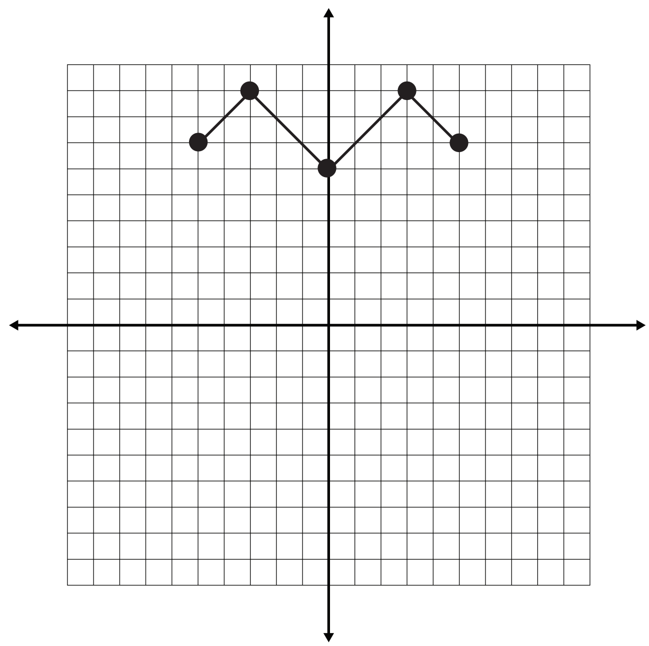
Example 6 Draw the graph resulting from each transformation.

Graphing Translations

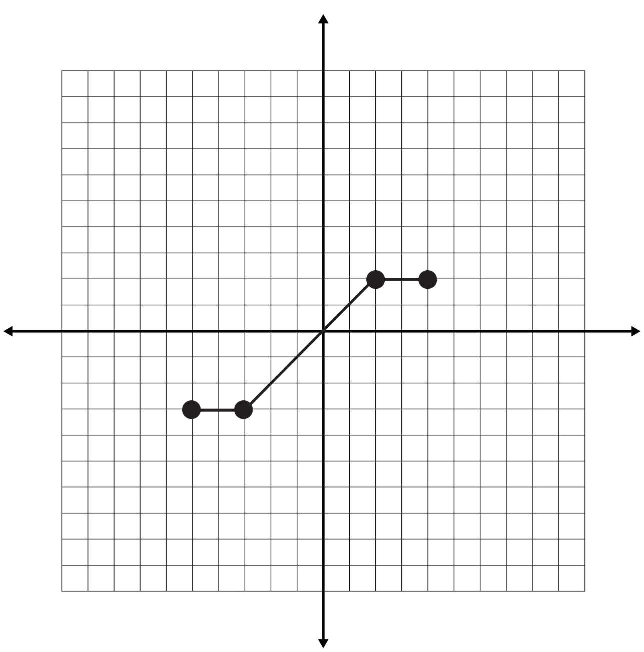
a) y - 4 = f(x)



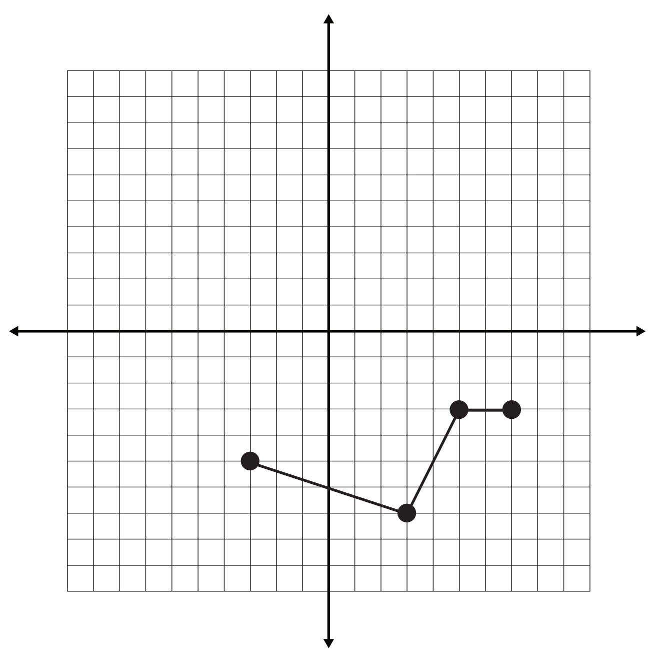
b) y = f(x) - 3

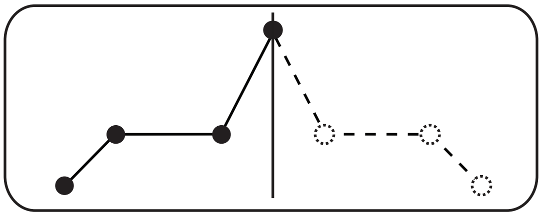


c) y = f(x - 5)



d) y = f(x + 4)



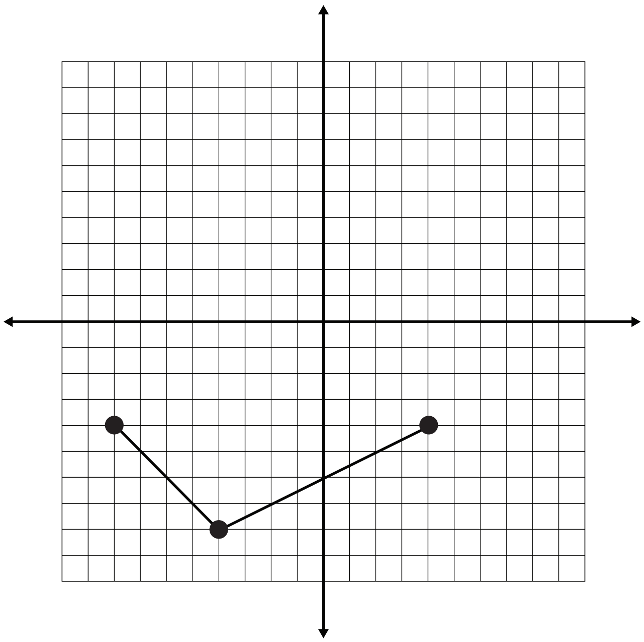


Transformations and Operations
LESSON ONE - *Basic Transformations*
Lesson Notes

Example 7 Draw the transformed graph. Write the transformation as both an equation and a mapping.

Mappings

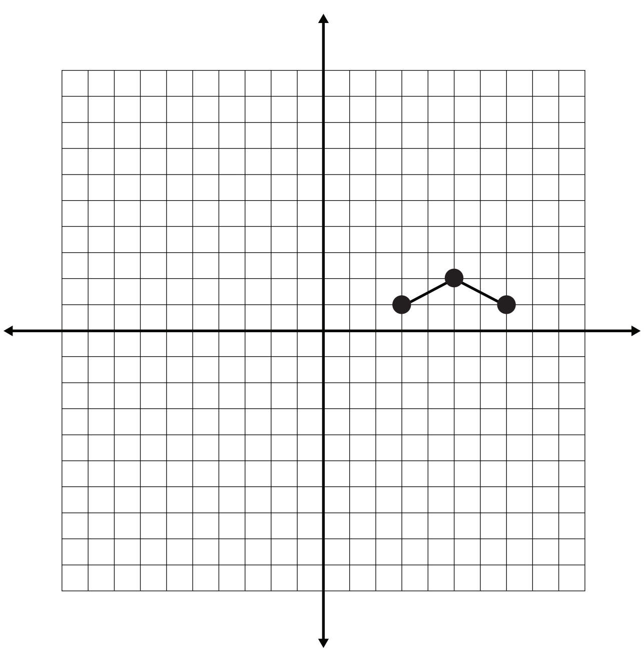
a) The graph of f(x) is horizontally stretched by a factor of $\frac{1}{2}$.



Transformation Equation:

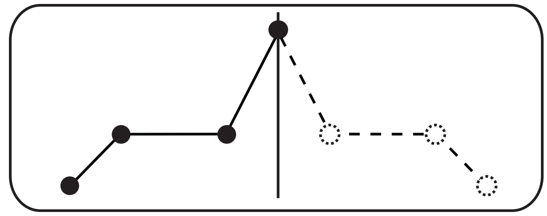
Transformation Mapping:

b) The graph of f(x) is horizontally translated 6 units left.

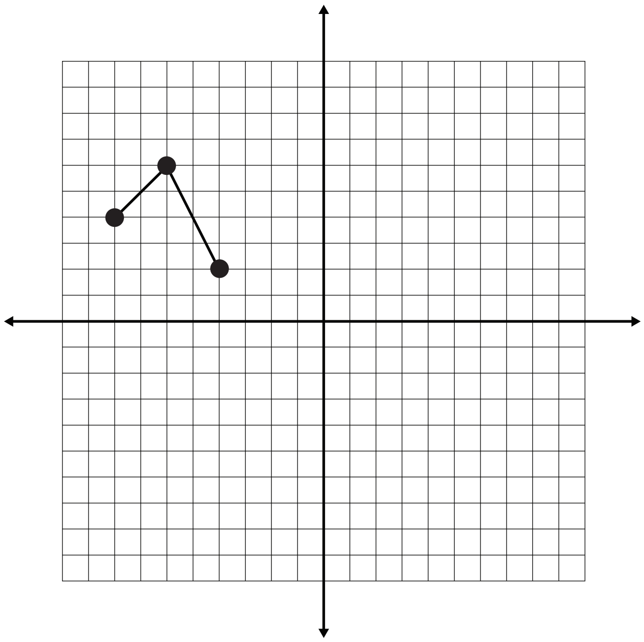


Transformation Equation:

Transformation Mapping:



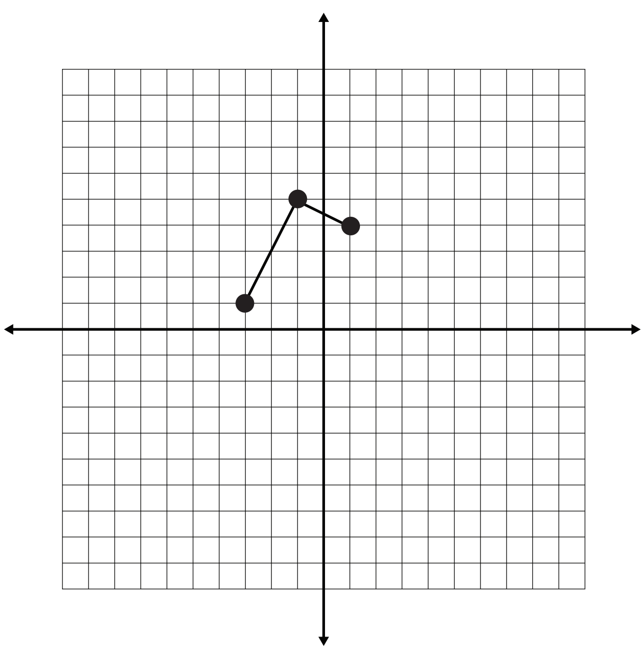
c) The graph of f(x) is vertically translated 4 units down.



Transformation Equation: ______

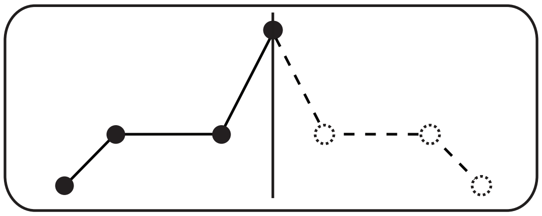
Transformation Mapping: ______

d) The graph of f(x) is reflected in the x-axis.



Transformation Equation: ______

Transformation Mapping: ______



Transformations and Operations
LESSON ONE - *Basic Transformations*
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Example 8 Write a sentence describing each transformation, then write the transformation equation.

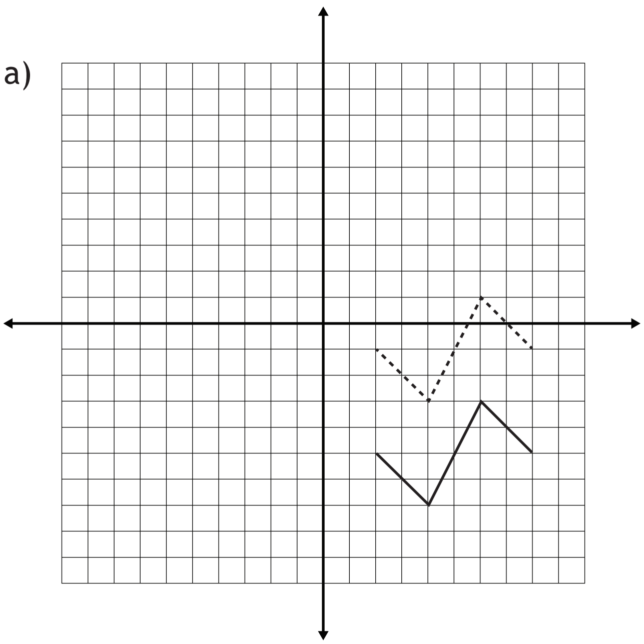
Describing a Transformation

Original graph: --------------

Transformed graph: ————

Think of the dashed line as representing where the graph was in the past, and the solid line is where the graph is now.

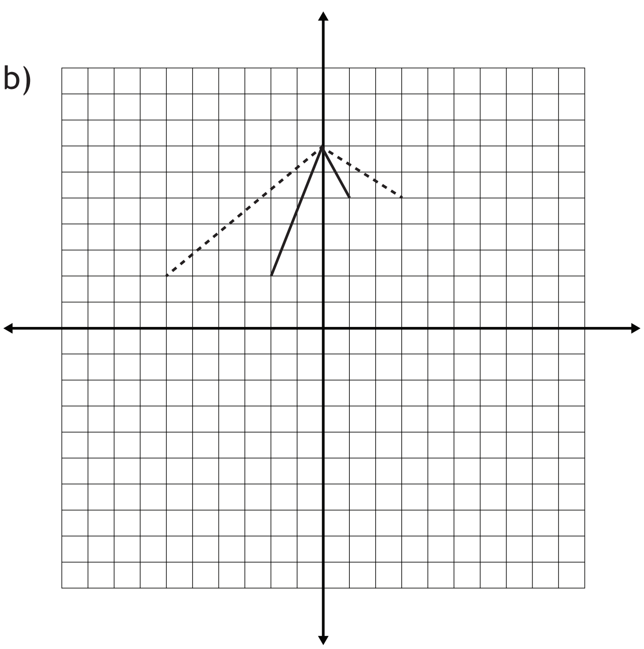
a)



Transformation Equation:

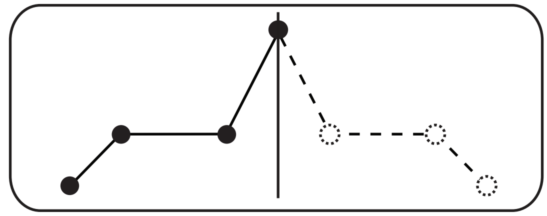
Transformation Mapping:

b)

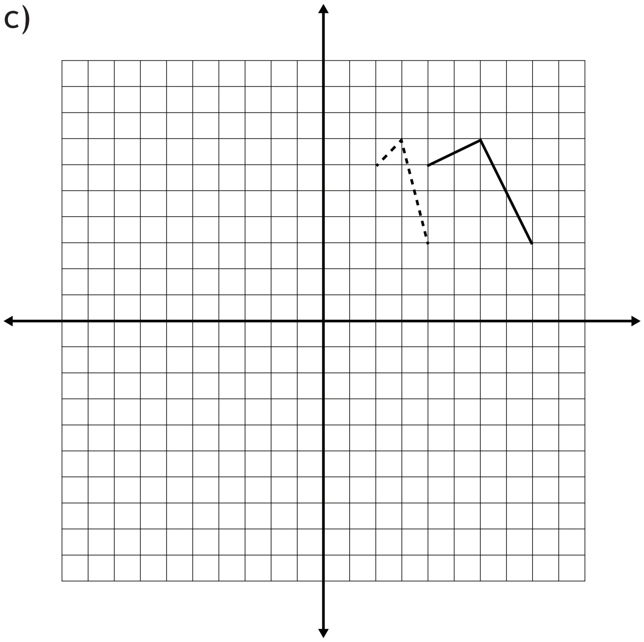


Transformation Equation:

Transformation Mapping:



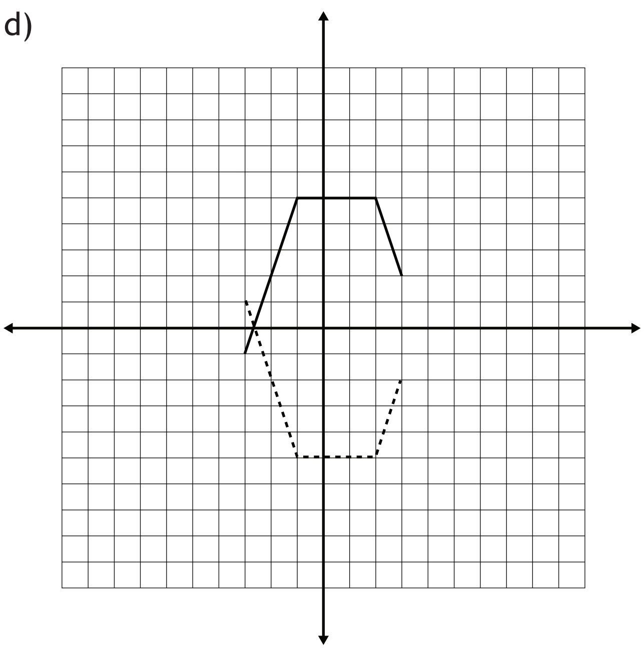
c)



Transformation Equation: ________

Transformation Mapping: ________

d)



Transformation Equation: ________

Transformation Mapping: ________

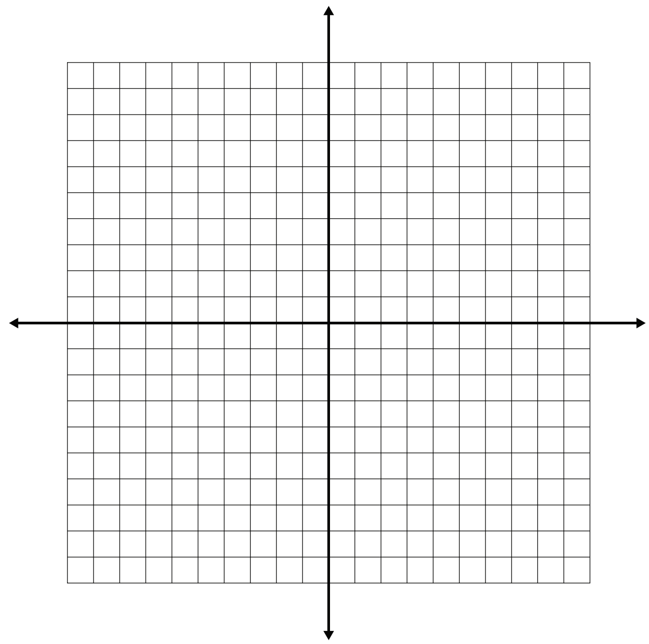
Example 9 Describe each transformation and derive the equation of the transformed graph. Draw the original and transformed graphs.

Transforming an Existing Function ***(stretches)***

a) Original graph: $f(x) = x^2 - 1$
Transformation: $y = 2f(x)$

Transformation Description:

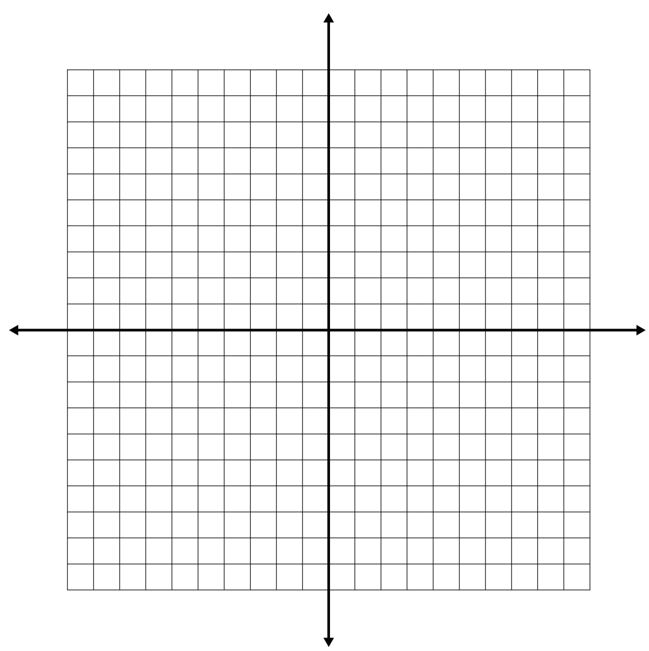
New Function After Transformation:

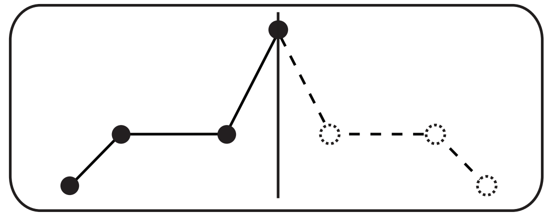


b) Original graph: $f(x) = x^2 + 1$
Transformation: $y = f(2x)$

Transformation Description:

New Function After Transformation:



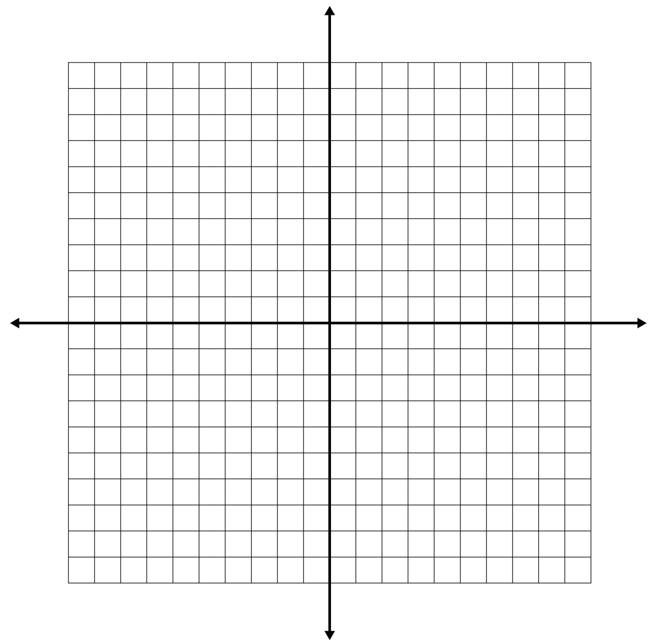


Transforming an Existing Function *(reflections)*

c) Original graph: $f(x) = x^2 - 2$
Transformation: $y = -f(x)$

Transformation Description:

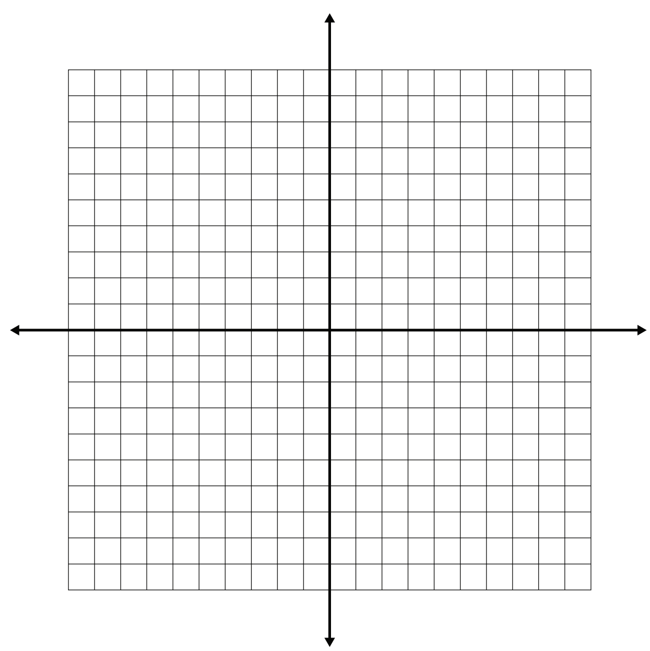
New Function After Transformation:



d) Original graph: $f(x) = (x - 6)^2$
Transformation: $y = f(-x)$

Transformation Description:

New Function After Transformation:



Example 10

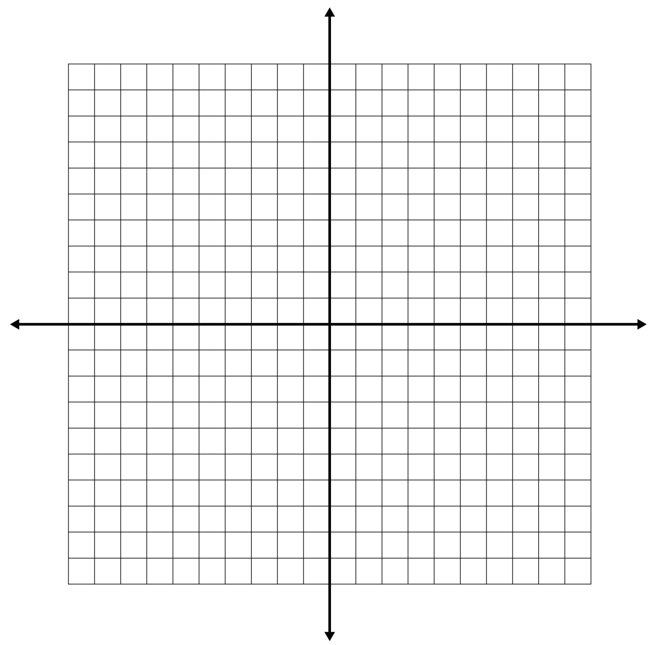
Describe each transformation and derive the equation of the transformed graph. Draw the original and transformed graphs.

Transforming an Existing Function *(translations)*

a) Original graph: $f(x) = x^2$
Transformation: $y - 2 = f(x)$

Transformation Description:

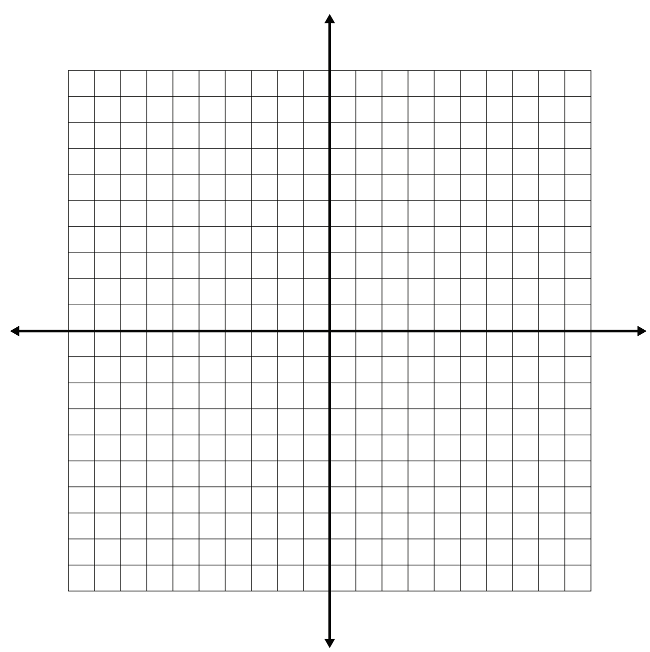
New Function After Transformation:



b) Original graph: $f(x) = x^2 - 4$
Transformation: $y = f(x) - 4$

Transformation Description:

New Function After Transformation:

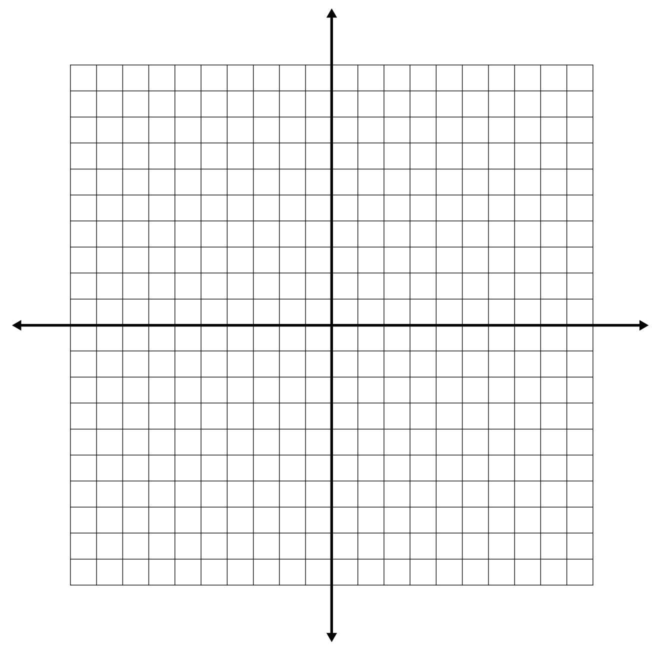


Transforming an Existing Function ***(translations)***

c) Original graph: $f(x) = x^2$
Transformation: $y = f(x - 2)$

Transformation Description:

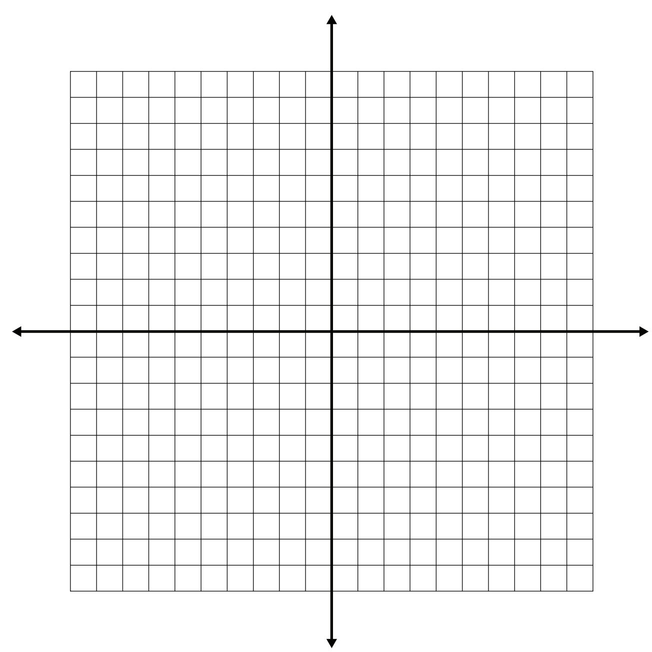
New Function After Transformation:



d) Original graph: $f(x) = (x + 3)^2$
Transformation: $y = f(x - 7)$

Transformation Description:

New Function After Transformation:

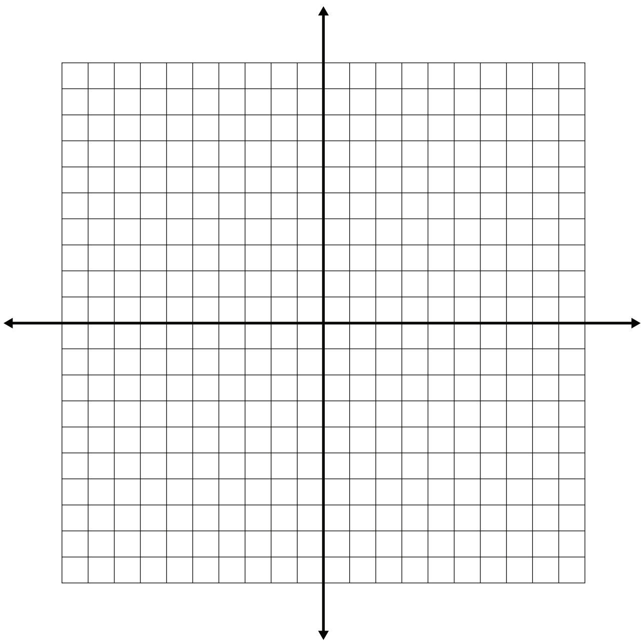


Example 11

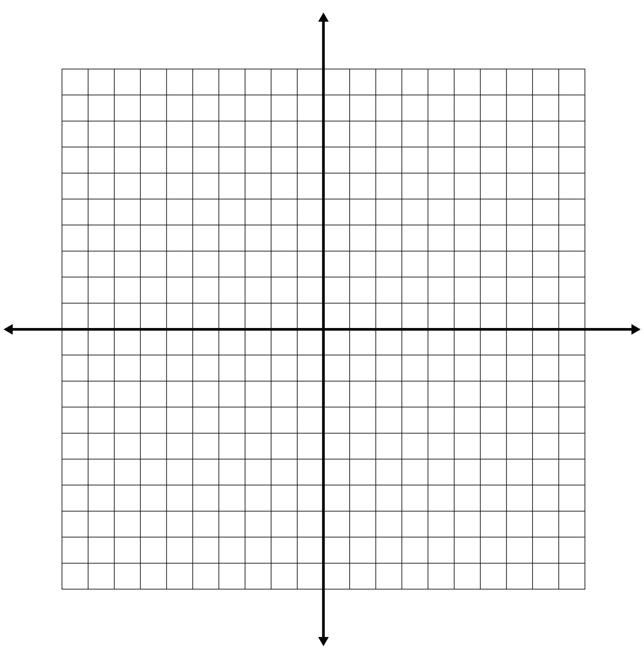
Answer the following questions:

What Transformation Occured?

a) The graph of $y = x^2 + 3$ is vertically translated so it passes through the point (2, 10).
Write the equation of the applied transformation. *Solve graphically first, then solve algebraically.*



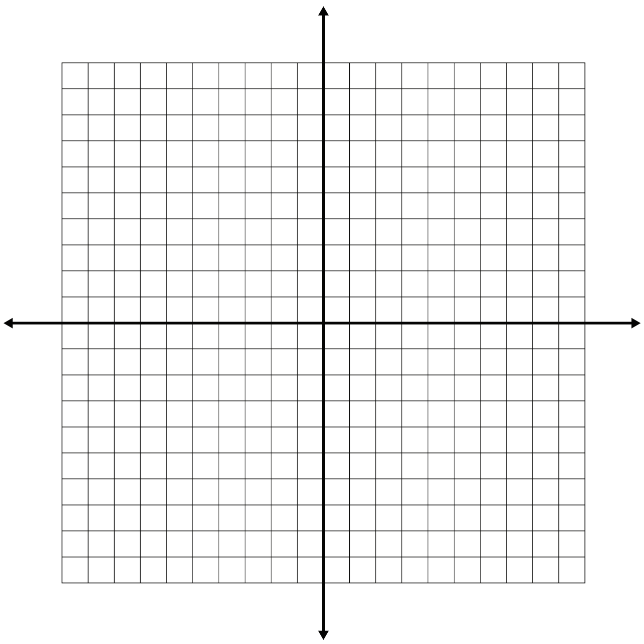
b) The graph of $y = (x + 2)^2$ is horizontally translated so it passes through the point (6, 9).
Write the equation of the applied transformation.
Solve graphically first, then solve algebraically.



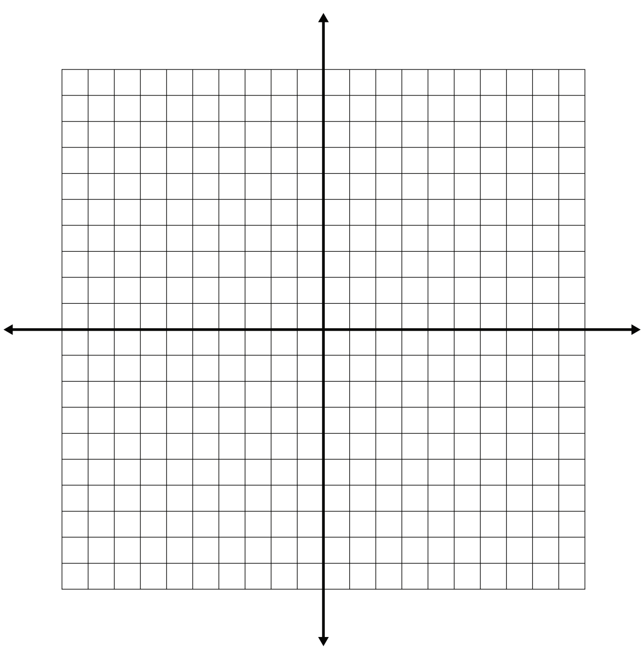
Example 12 Answer the following questions:

What Transformation Occured?

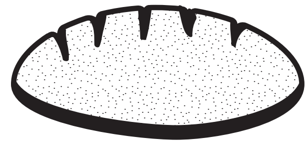
a) The graph of $y = x^2 - 2$ is vertically stretched so it passes through the point (2, 6). Write the equation of the applied transformation. *Solve graphically first, then solve algebraically.*



b) The graph of $y = (x - 1)^2$ is transformed by the equation $y = f(bx)$. The transformed graph passes through the point (-4, 4). Write the equation of the applied transformation.
Solve graphically first, then solve algebraically.

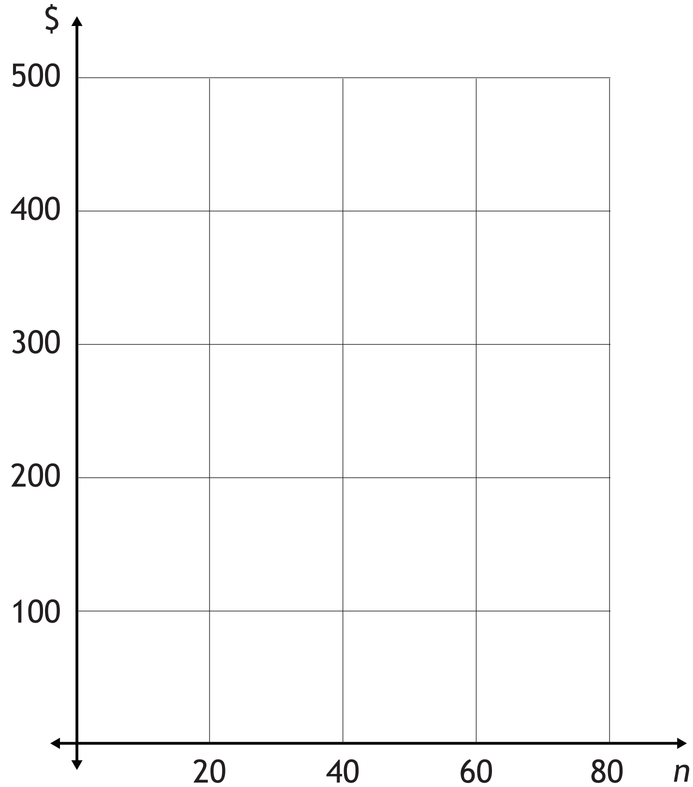


Example 13



Sam sells bread at a farmers' market for $5.00 per loaf. It costs $150 to rent a table for one day at the farmers' market, and each loaf of bread costs $2.00 to produce.

a) Write two functions, R(n) and C(n), to represent Sam's revenue and costs. Graph each function.

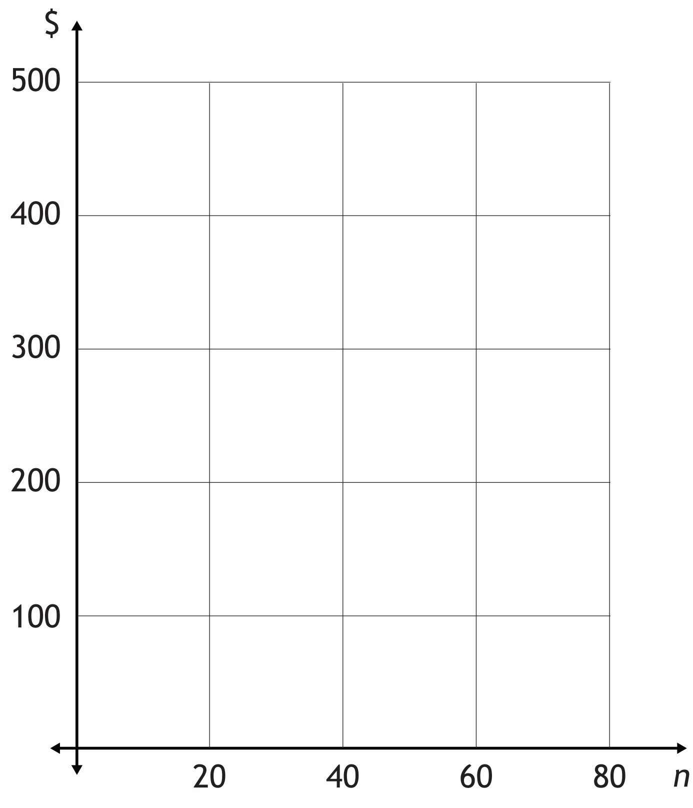


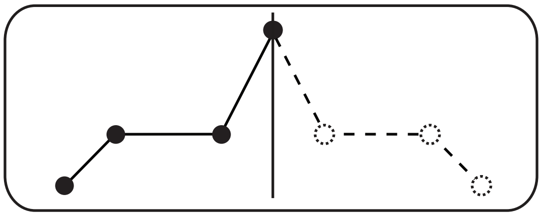
b) How many loaves of bread does Sam need to sell in order to make a profit?

c) The farmers' market raises the cost of renting a table by $50 per day. Use a transformation to find the new cost function, $C_2(n)$.

d) In order to compensate for the increase in rental costs, Sam will increase the price of a loaf of bread by 20%. Use a transformation to find the new revenue function, $R_2(n)$.

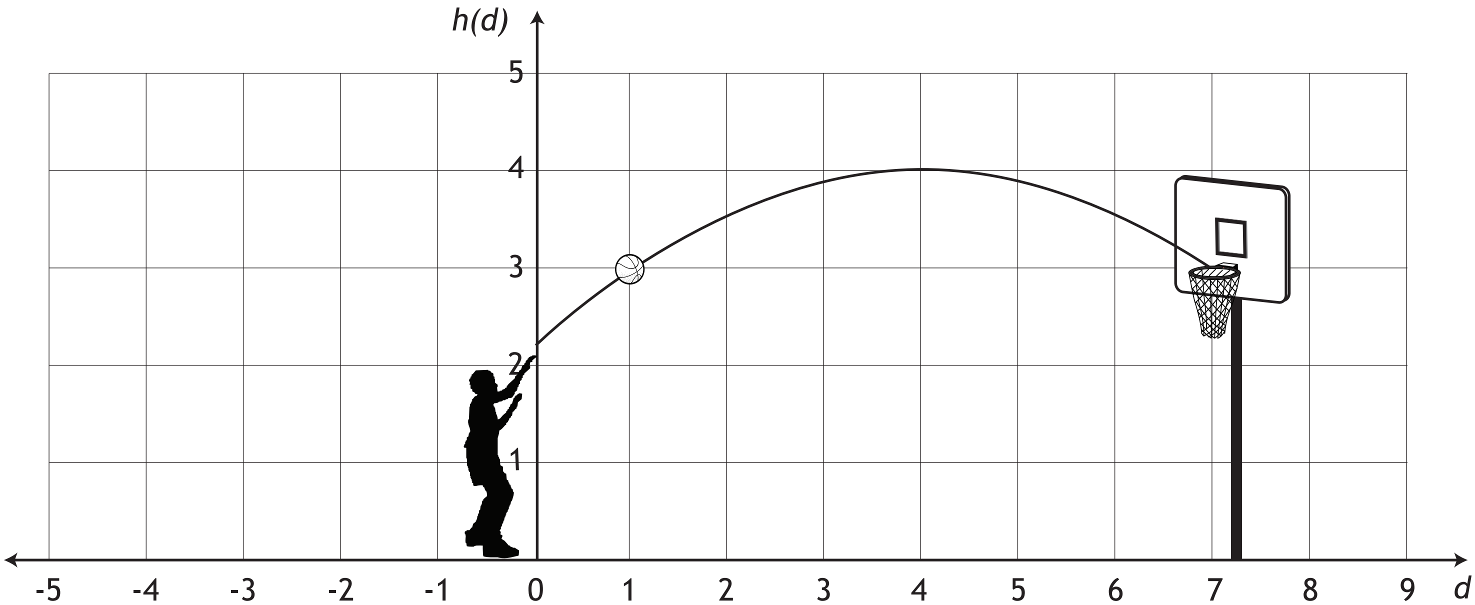
e) Draw the transformed functions from parts (c) and (d). How many loaves of bread does Sam need to sell now in order to break even?





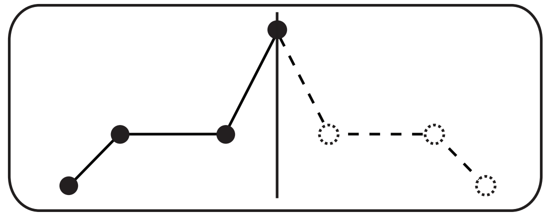
Example 14

A basketball player throws a basketball. The path can be modeled with $h(d) = -\frac{1}{9}(d - 4)^2 + 4$.



a) Suppose the player moves 2 m closer to the hoop before making the shot. Determine the equation of the transformed graph, draw the graph, and predict the outcome of the shot.

b) If the player moves so the equation of the shot is $h(d) = -\frac{1}{9}(d + 1)^2 + 4$, what is the horizontal distance from the player to the hoop?



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Example 1

Combined Transformations

Combining Stretches and Reflections

a) Identify each parameter in the general transformation equation: $y = af[b(x - h)] + k$.

b) Describe the transformations in each equation:

i) $y = \frac{1}{3}f(5x)$

ii) $y = 2f(\frac{1}{4}x)$

iii) $y = -\frac{1}{2}f(\frac{1}{3}x)$

iv) $y = -3f(-2x)$

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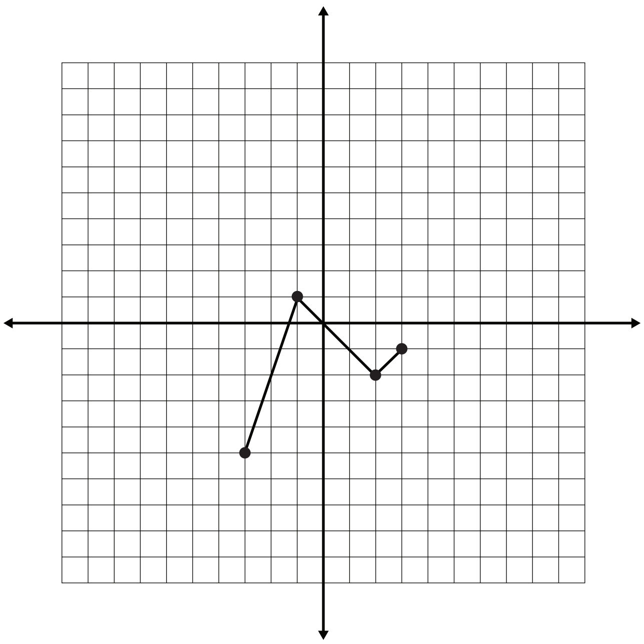
$y = af[b(x - h)] + k$

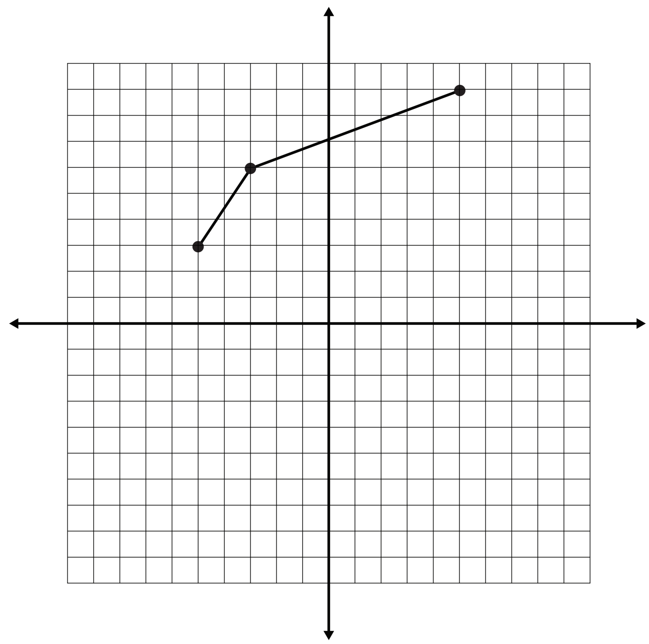
Example 2 Draw the transformation of each graph.

Combining Stretches and Reflections

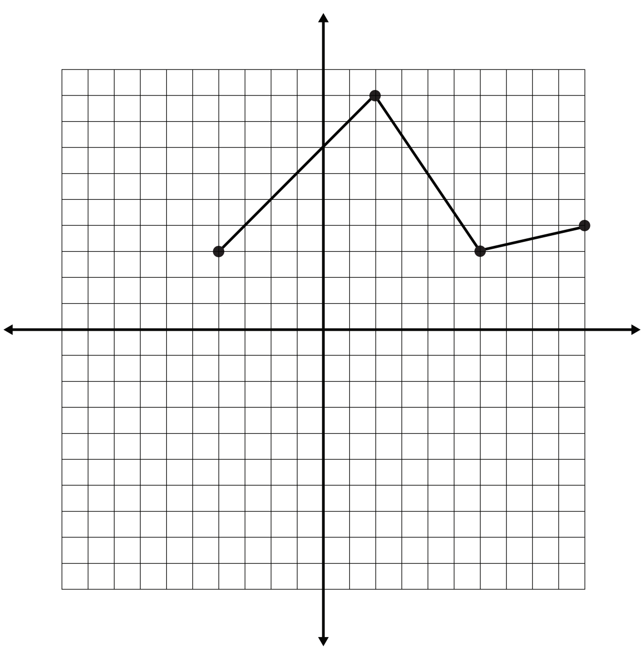
a) $y = 2f(\frac{1}{3}x)$



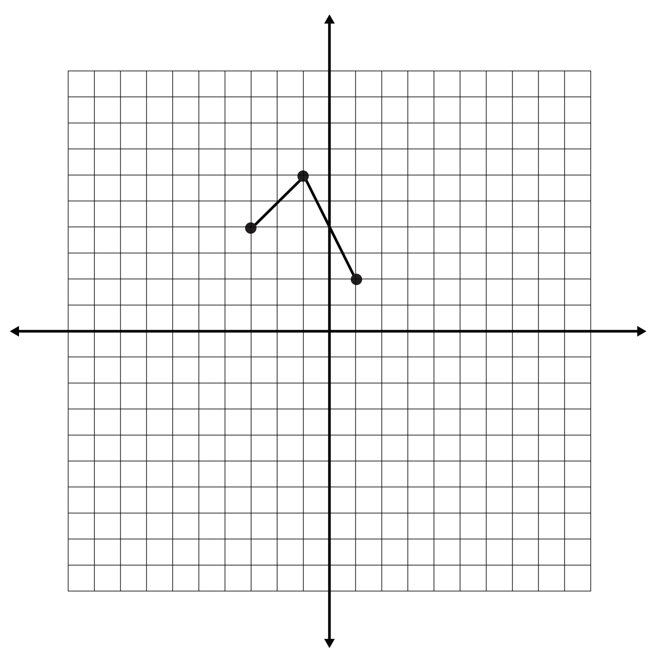
b) $y = \frac{1}{3}f(-x)$



c) $y = -f(2x)$



d) $y = -\frac{1}{2}f(-x)$



Example 3

Answer the following questions:

Combining Translations

a) Find the horizontal translation of $y = f(x + 3)$ using three different methods.

Opposite Method: Zero Method: Double Sign Method:

b) Describe the transformations in each equation:

i) $y = f(x - 1) + 3$

ii) $y = f(x + 2) - 4$

iii) $y = f(x - 2) - 3$

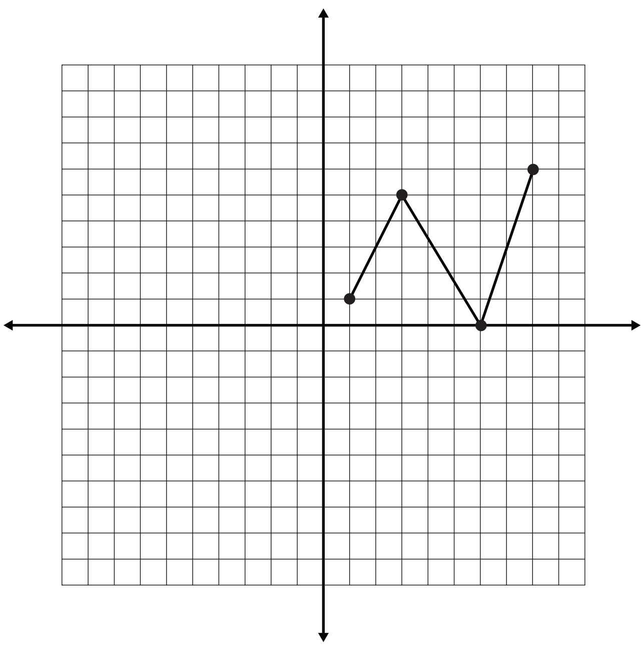
iv) $y = f(x + 7) + 5$

$y = af[b(x - h)] + k$

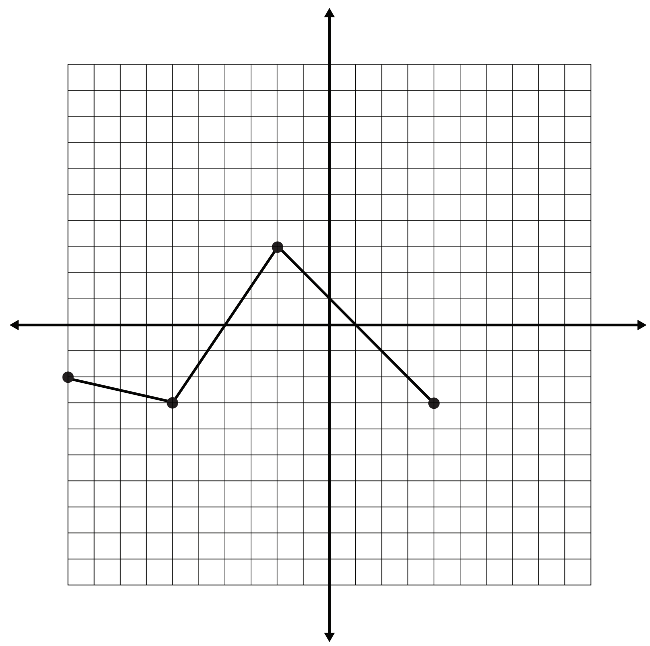
Example 4 Draw the transformation of each graph.

Combining Translations

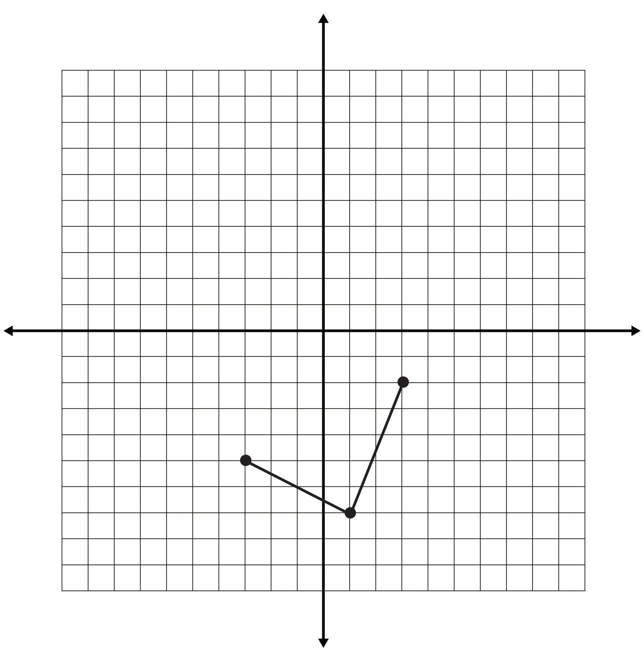
a) y = f(x + 5) - 3



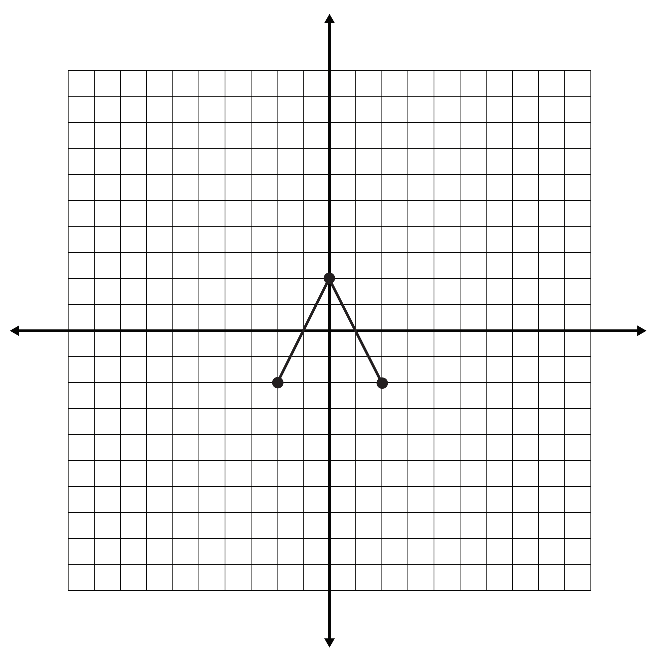
b) y = f(x - 3) + 7



c) y - 12 = f(x - 6)



d) y + 2 = f(x + 8)



Example 5 Answer the following questions:

Combining Stretches, Reflections, and Translations

a) When applying transformations to a graph, should they be applied in a specific order?

b) Describe the transformations in each equation.

i) $y = 2f(x + 3) + 1$

ii) $y = -f(\frac{1}{3}x) - 4$

iii) $y = \frac{1}{2}f[-(x + 2)] - 3$

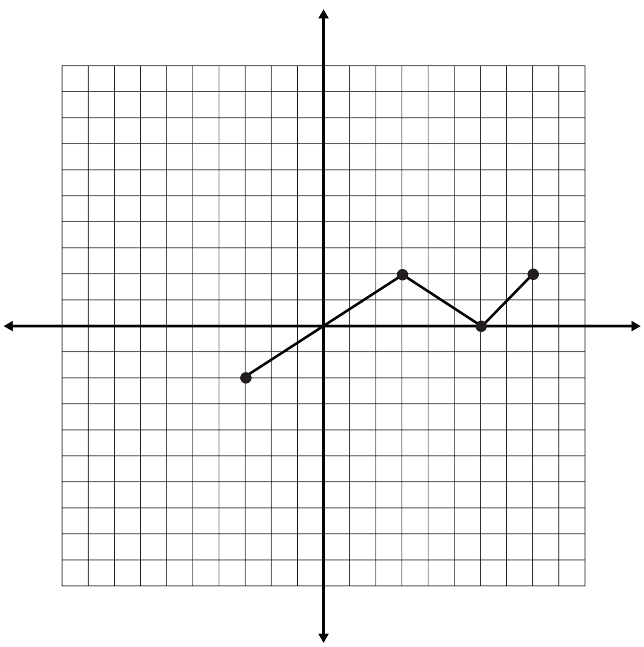
iv) $y = -3f[-4(x - 1)] + 2$

$$y = af[b(x - h)] + k$$

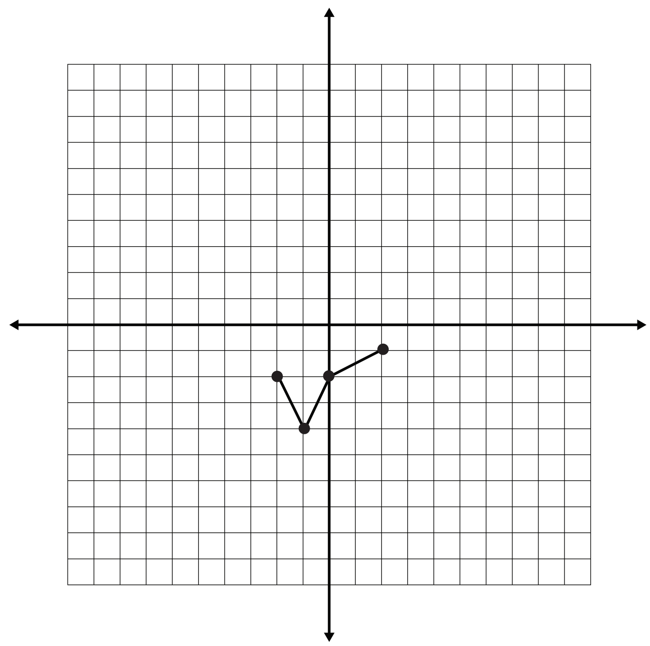
Example 6 Draw the transformation of each graph.

Combining Stretches, Reflections, and Translations

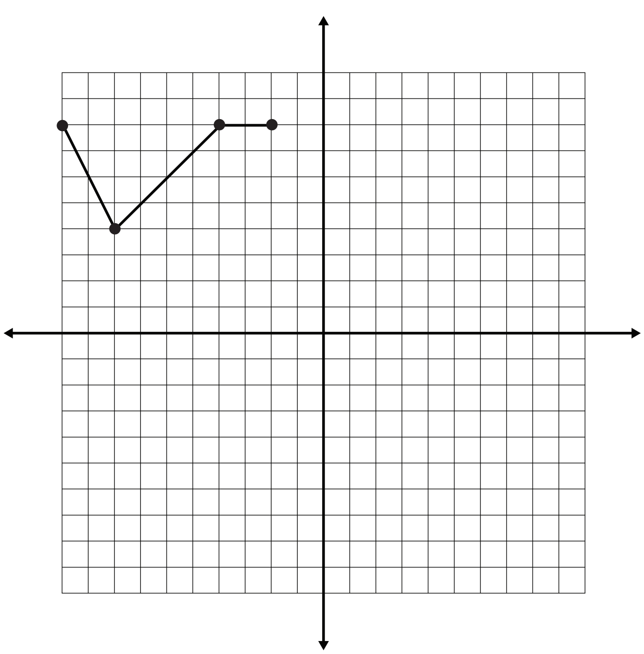
a) $y = -f(x) - 2$



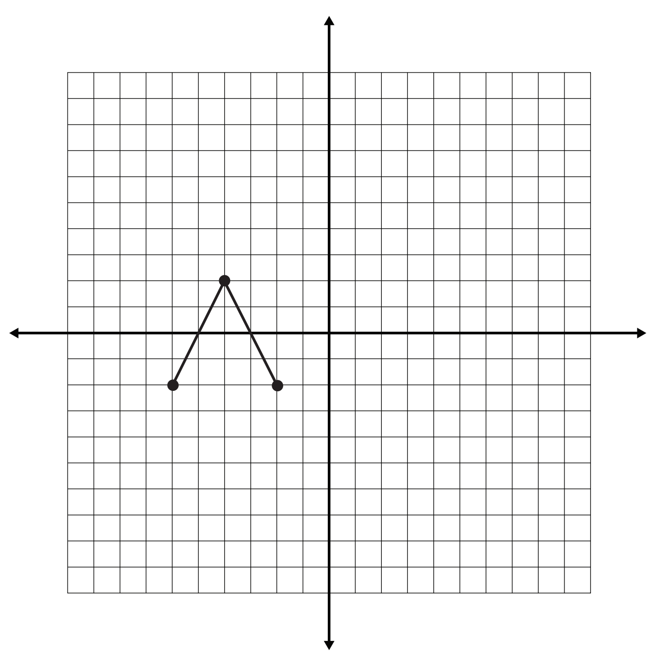
b) $y = f(-\frac{1}{4}x) + 1$



c) $y = -\frac{1}{4}f(2x) - 1$



d) $2y - 8 = 6f(x - 2)$



$y = af[b(x - h)] + k$

Transformations and Operations
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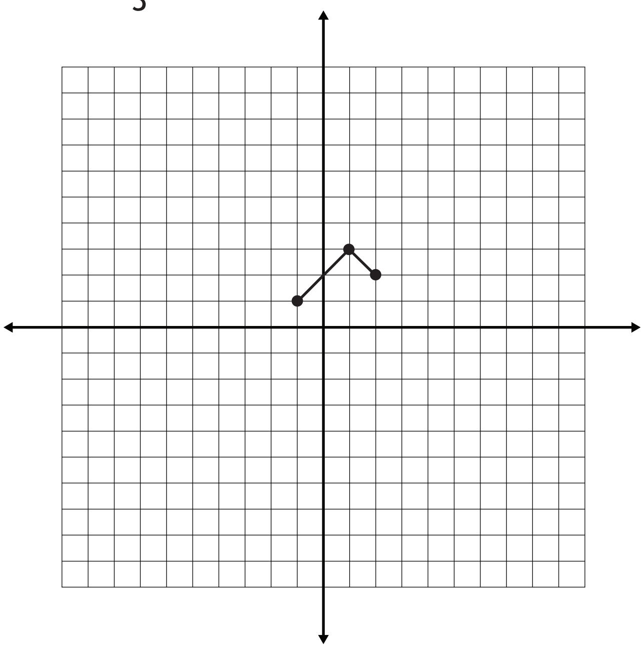
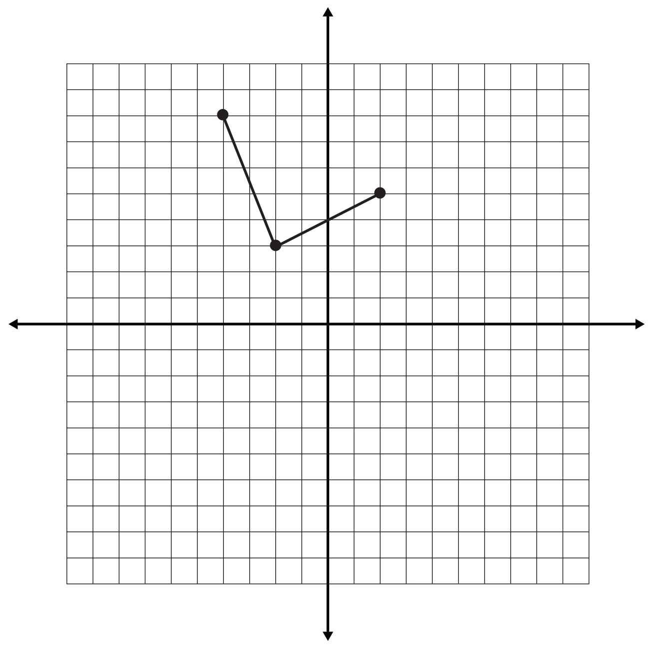
Example 7 Draw the transformation of each graph.

Combining Stretches, Reflections, and Translations
(watch for b-factoring!)

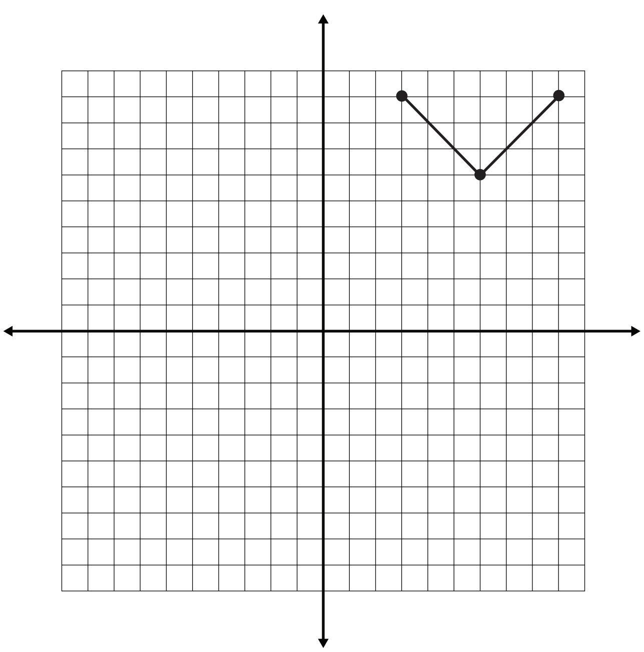
a) $y = f[\frac{1}{3}(x - 1)] + 1$



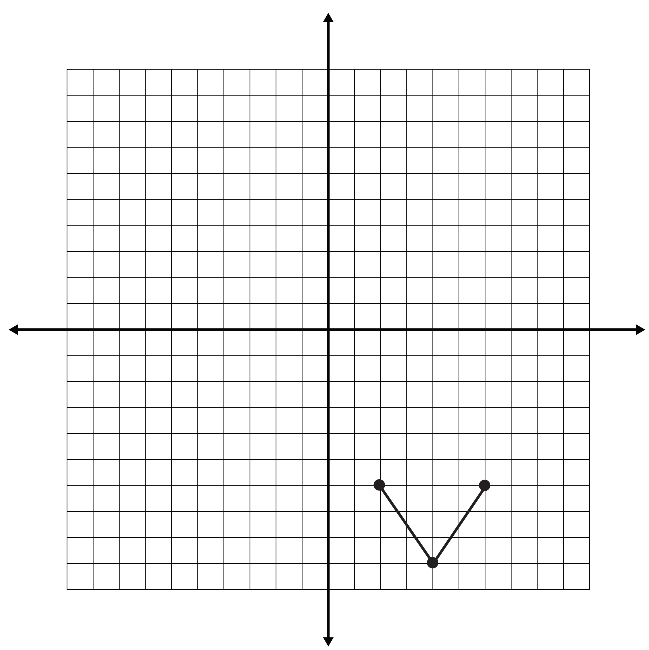
b) $y = f(2x + 6)$



c) $y = f(3x - 6) - 2$



d) $y = \frac{1}{3}f(-x - 4)$



$$y = af[b(x - h)] + k$$

Example 8 Answer the following questions:

Mappings

The mapping for combined transformations is:

$$(x, y) \rightarrow \left(\frac{x_i}{b} + h,\ ay_i + k\right)$$

a) If the point (2, 0) exists on the graph of y = f(x), find the coordinates of the new point after the transformation y = f(-2x + 4).

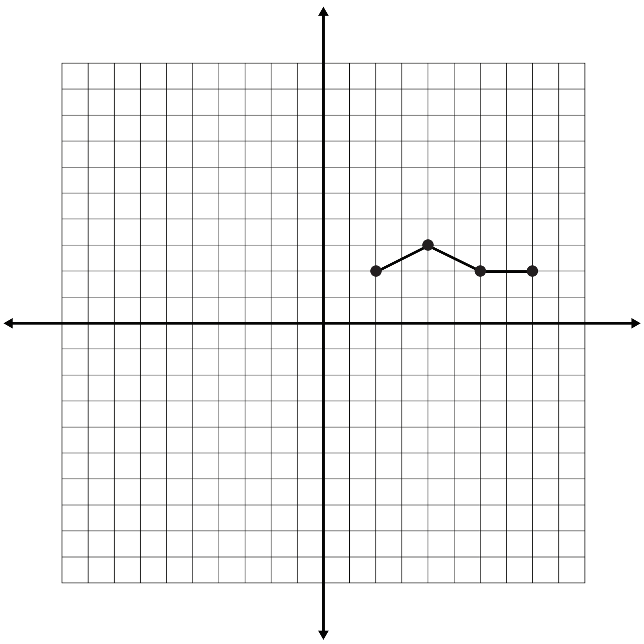
b) If the point (5, 4) exists on the graph of y = f(x), find the coordinates of the new point after the transformation $y = \frac{1}{2}f(5x - 10) + 4$.

c) The point (m, n) exists on the graph of y = f(x). If the transformation y = 2f(2x) + 5 is applied to the graph, the transformed point is (4, 7). Find the values of m and n.

Example 9 For each transformation description, write the transformation equation. Use mappings to draw the transformed graph.

Mappings

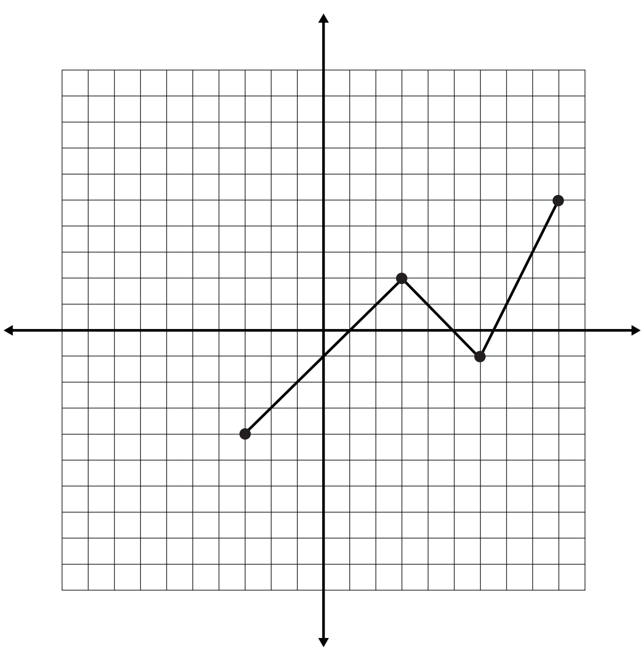
a) The graph of y = f(x) is vertically stretched by a factor of 3, reflected about the x-axis, and translated 2 units to the right.



Transformation Equation:

Mappings:

b) The graph of y = f(x) is horizontally stretched by a factor of $\frac{1}{3}$, reflected about the x-axis, and translated 2 units left.



Transformation Equation:

Mappings:

Transformations and Operations
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$y = af[b(x - h)] + k$

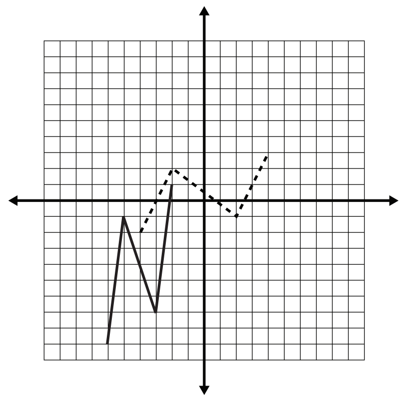
Example 10 Order of Transformations. Axis-Independence

Greg applies the transformation y = -2f[-2(x + 4)] - 3 to the graph below, using the transformation order rules learned in this lesson.

Greg's Transformation Order:

Stretches & Reflections:
1) Vertical stretch by a scale factor of 2
2) Reflection about the x-axis
3) Horizontal stretch by a scale factor of 1/2
4) Reflection about the y-axis

Translations:
5) Vertical translation 3 units down
6) Horizontal translation 4 units left



Original graph: - - - - - - - - - - - - - -

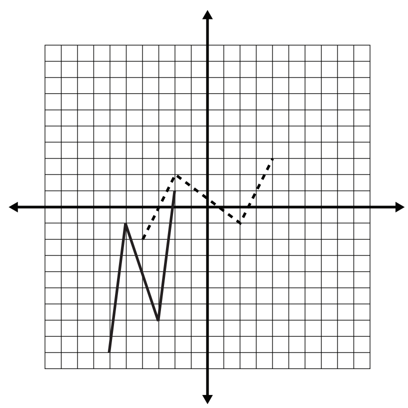
Transformed graph: ________

Next, Colin applies the same transformation, y = -2f[-2(x + 4)] - 3, to the graph below.
He tries a different transformation order, applying all the vertical transformations first, followed by all the horizontal transformations.

Colin's Transformation Order:

Vertical Transformations:
1) Vertical stretch by a scale factor of 2
2) Reflection about the x-axis
3) Vertical translation 3 units down.

Horizontal Transformations:
4) Horizontal stretch by a scale factor of 1/2
5) Reflection about the y-axis
6) Horizontal translation 4 units left

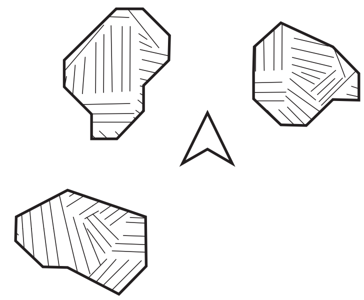


Original graph: - - - - - - - - - - - - - -

Transformed graph: ________

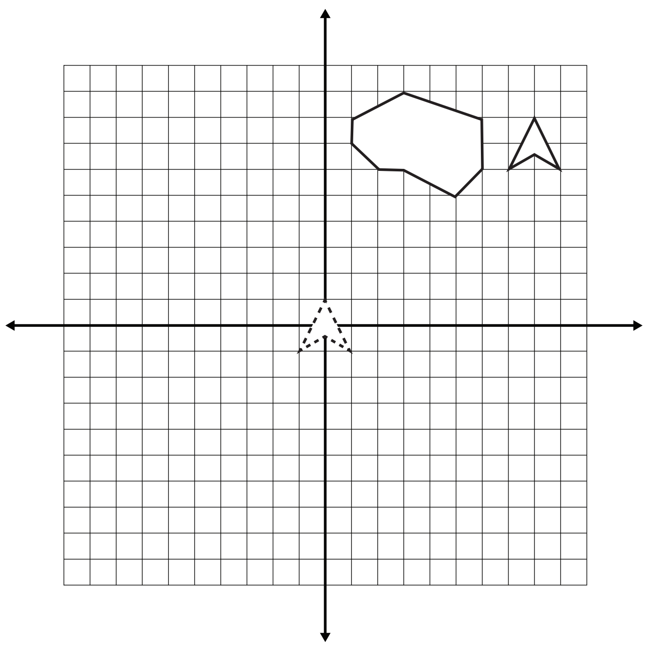
According to the transformation order rules we have been using in this lesson *(stretches & reflections first, translations last)*, Colin should obtain the wrong graph.
However, Colin obtains the same graph as Greg! How is this possible?

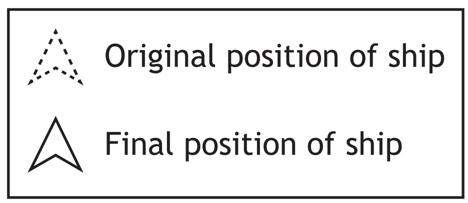
Example 11



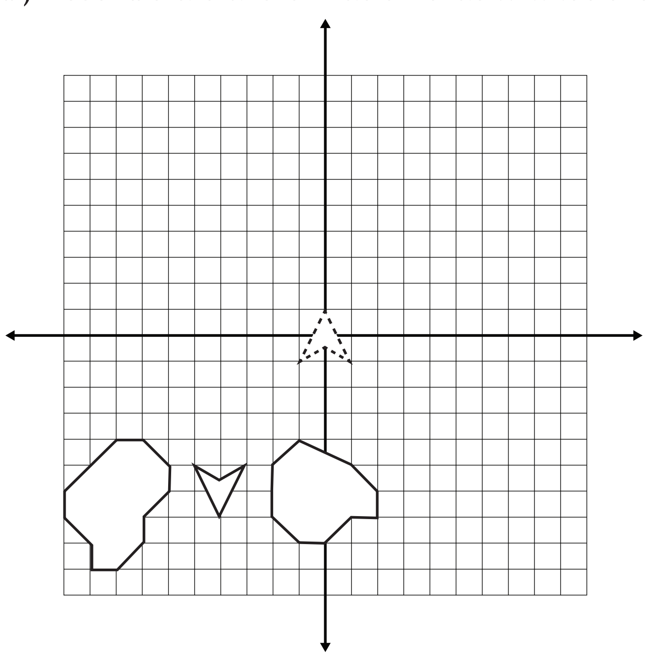
The goal of the video game *Space Rocks* is to pilot a spaceship through an asteroid field without colliding with any of the asteroids.

a) If the spaceship avoids the asteroid by navigating to the position shown, describe the transformation.



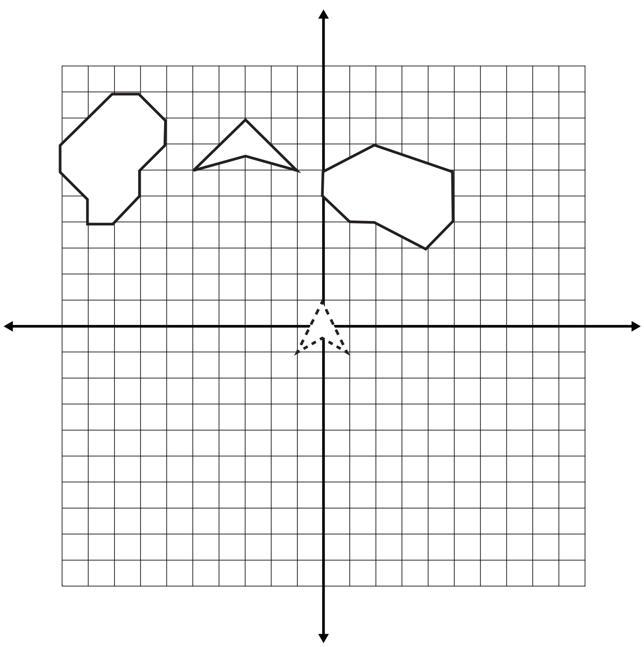


b) Describe a transformation that will let the spaceship pass through the asteroids.

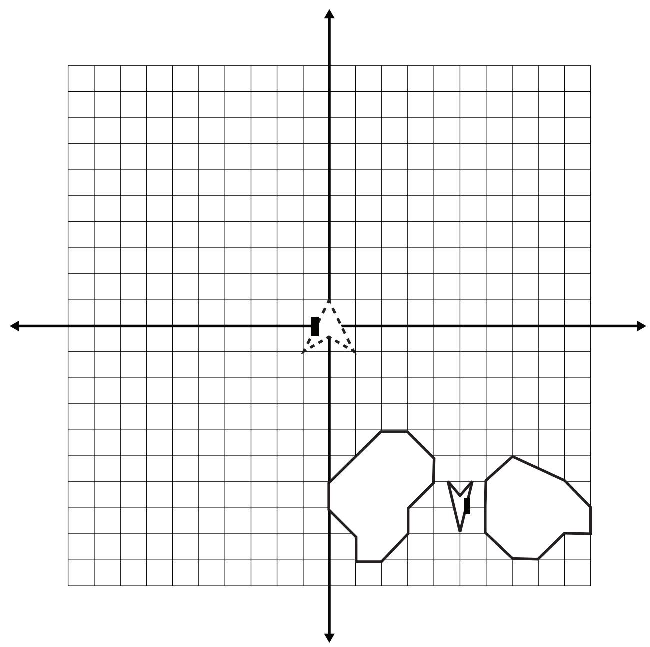


$y = af[b(x - h)] + k$

c) The spaceship acquires a power-up that gives it greater speed, but at the same time doubles its width. What transformation is shown in the graph?



d) The spaceship acquires two power-ups. The first power-up halves the original width of the spaceship, making it easier to dodge asteroids. The second power-up is a left wing cannon. What transformation describes the spaceship's new size and position?



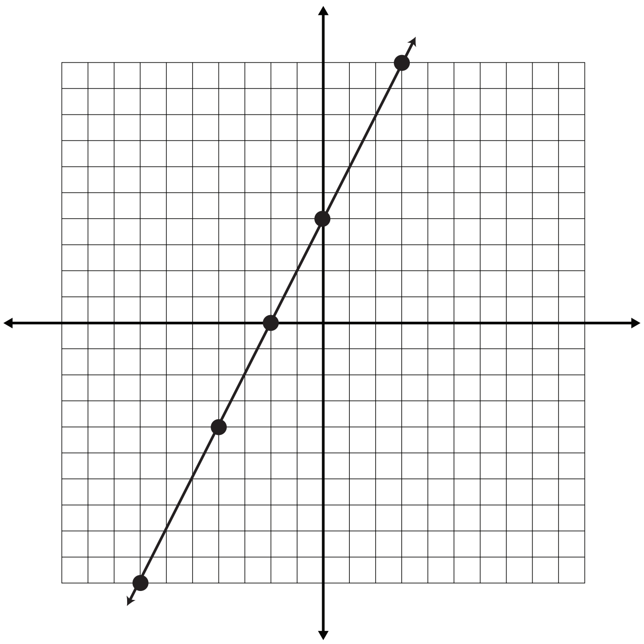
e) The transformations in parts (a - d) may **not** be written using $y = af[b(x - h)] + k$. Give two reasons why.

Example 1

Inverse Functions.

Finding an Inverse
(graphically and algebraically)

a) Given the graph of y = 2x + 4, draw the graph of the inverse.
What is the equation of the line of symmetry?



Inverse Mapping: (x, y) ⟶ (y, x)

(-7, -10) ⟶

(-4, -4) ⟶

(-2, 0) ⟶

(0, 4) ⟶

(3, 10) ⟶

b) Find the inverse function algebraically.

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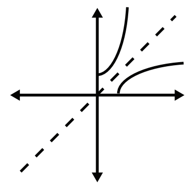
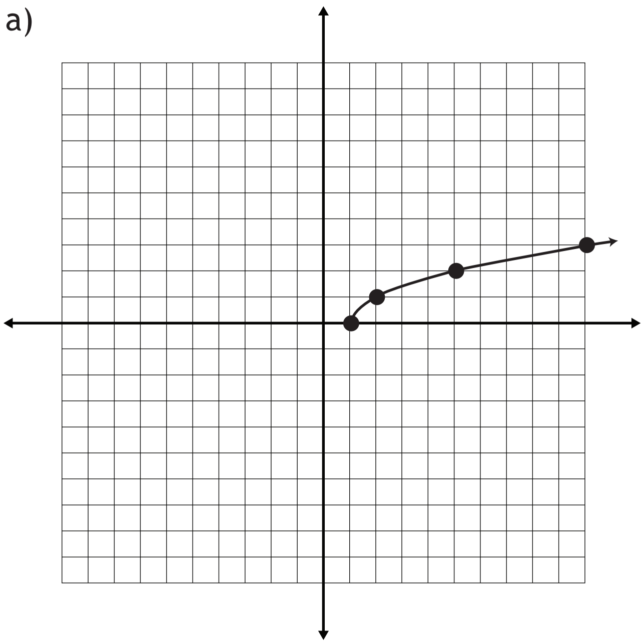


$f^{-1}(x)$

Example 2 For each graph, answer parts (i - iv).

Domain and Range

a)

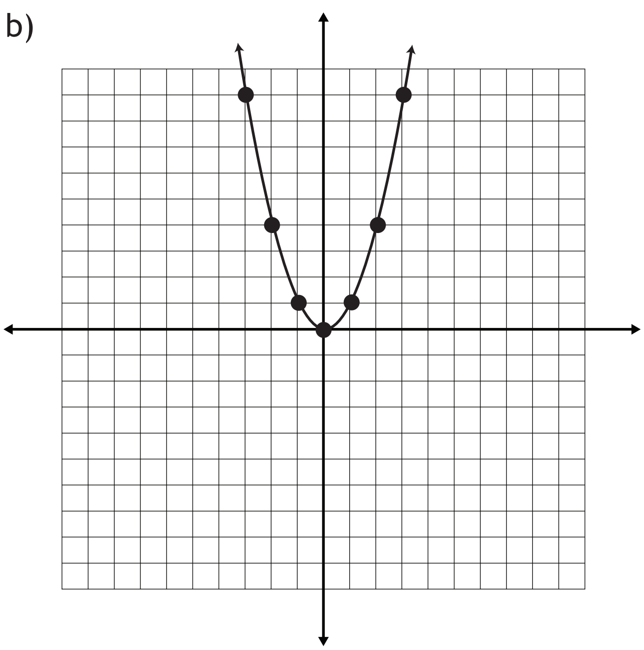


i) Draw the graph of the inverse.
ii) State the domain and range of the original graph.

iii) State the domain and range of the inverse graph.

iv) Can the inverse be represented with $f^{-1}(x)$?

b)

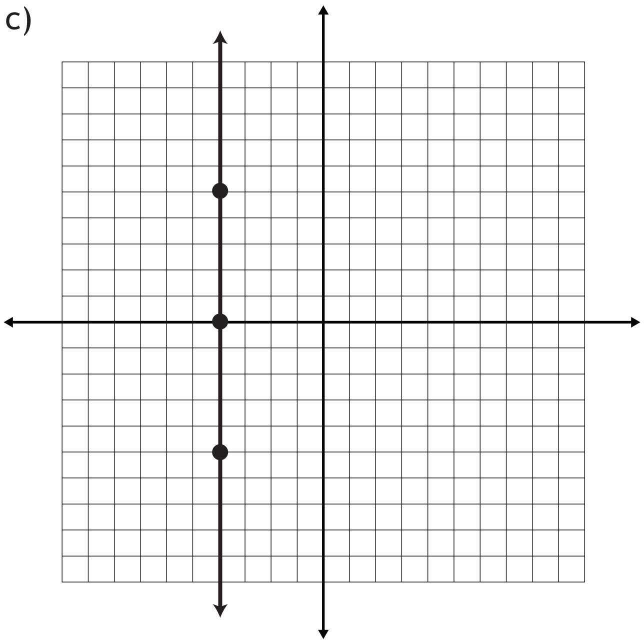


i) Draw the graph of the inverse.
ii) State the domain and range of the original graph.

iii) State the domain and range of the inverse graph.

iv) Can the inverse be represented with $f^{-1}(x)$?

c)

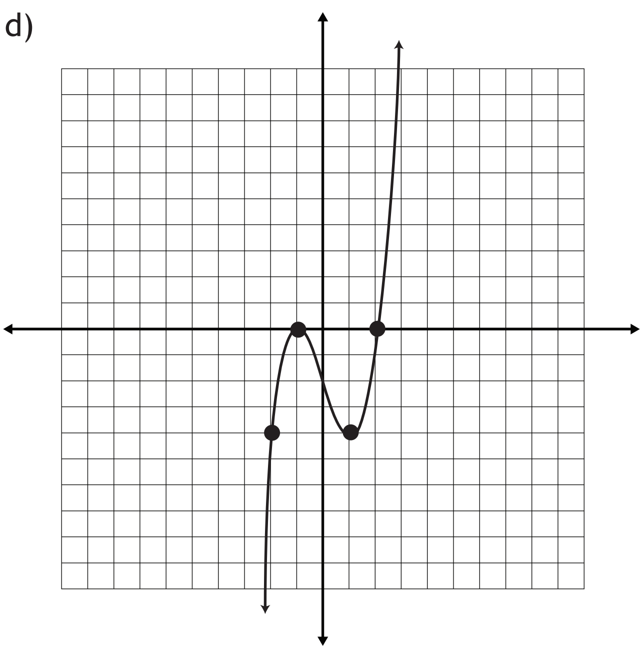


i) Draw the graph of the inverse.
ii) State the domain and range of the original graph.

iii) State the domain and range of the inverse graph.

iv) Can the inverse be represented with $f^{-1}(x)$?

d)



i) Draw the graph of the inverse.
ii) State the domain and range of the original graph.

iii) State the domain and range of the inverse graph.

iv) Can the inverse be represented with $f^{-1}(x)$?

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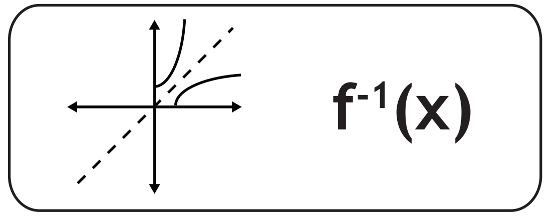
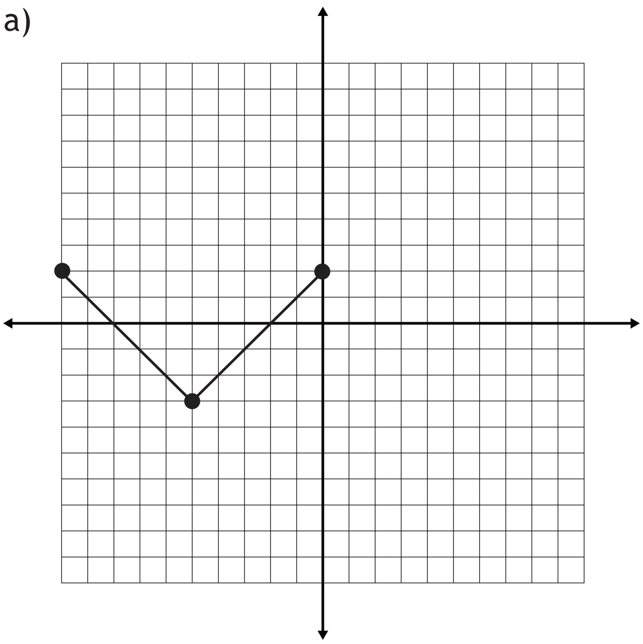


Example 3

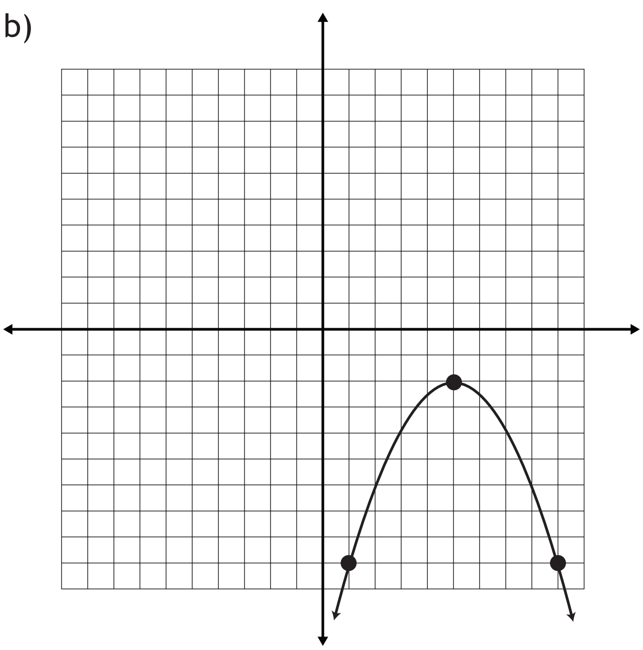
For each graph, draw the inverse. How should the domain of the original graph be restricted so the inverse is a function?

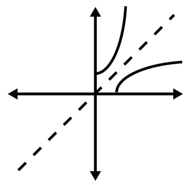
Domain Restrictions

a)



b)



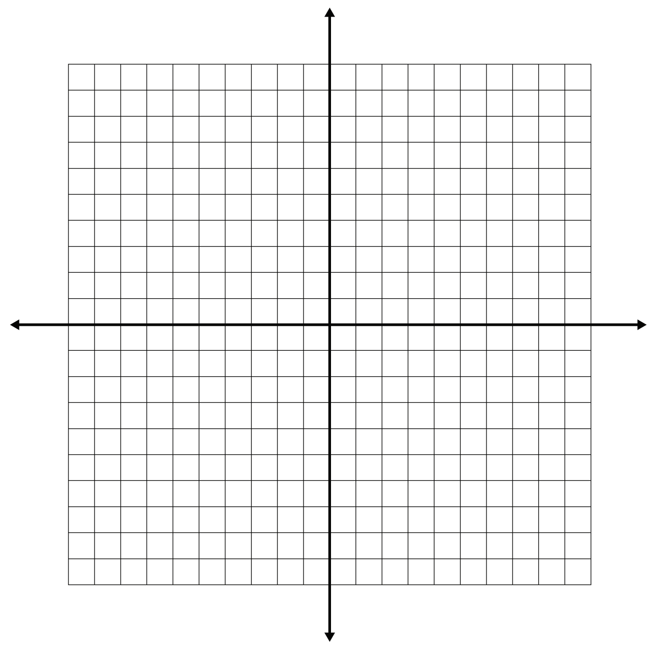


Example 4

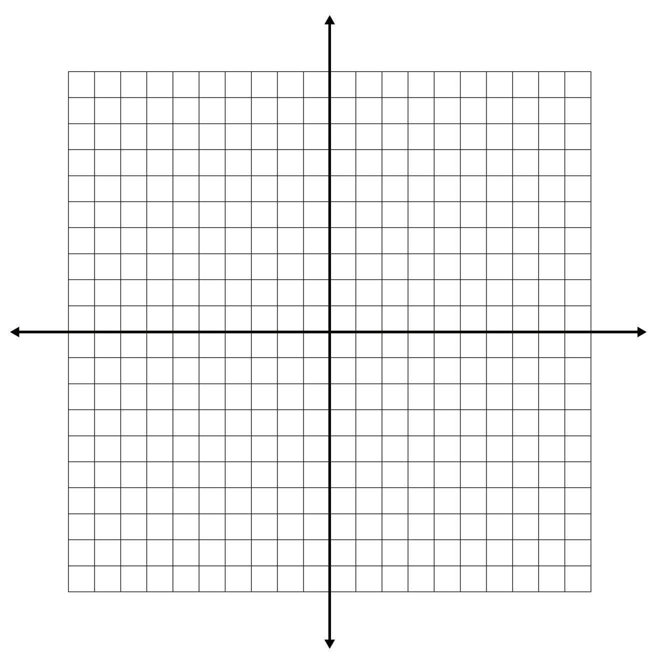
Find the inverse of each linear function algebraically. Draw the graph of the original function and the inverse. State the domain and range of both f(x) and its inverse.

Inverses of Linear Functions

a) $f(x) = x - 3$



b) $f(x) = -\frac{1}{2}x - 4$



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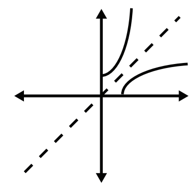


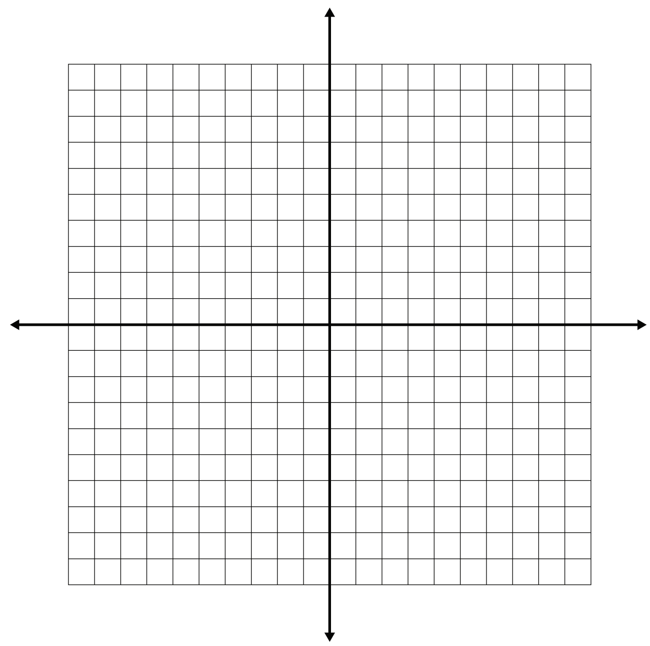
$f^{-1}(x)$

Example 5

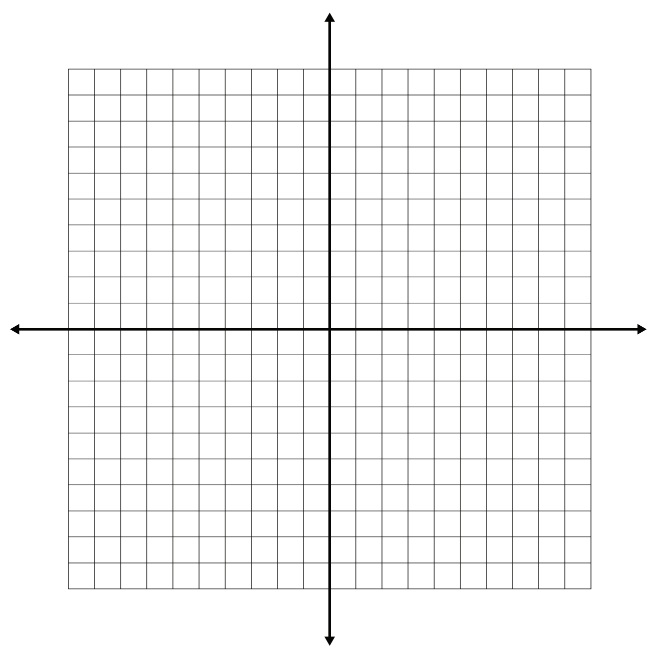
Find the inverse of each quadratic function algebraically. Draw the graph of the original function and the inverse. Restrict the domain of f(x) so the inverse is a function.

Inverses of Quadratic Functions

a) $f(x) = x^2 - 4$



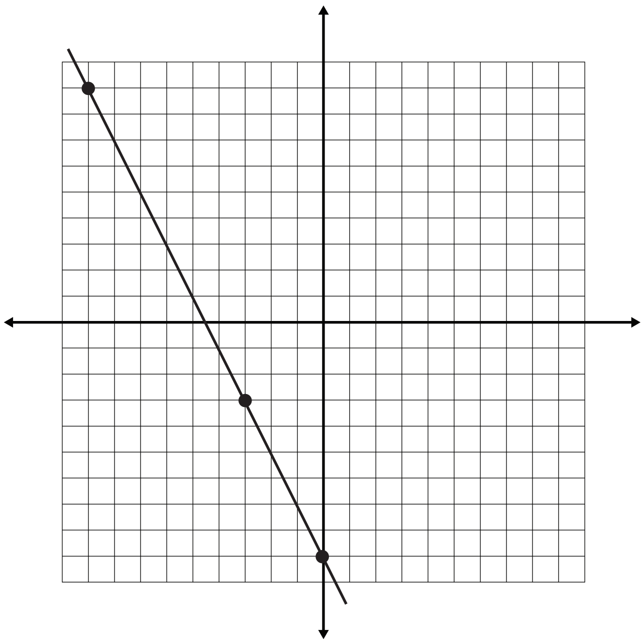
b) $f(x) = -(x + 3)^2 + 1$



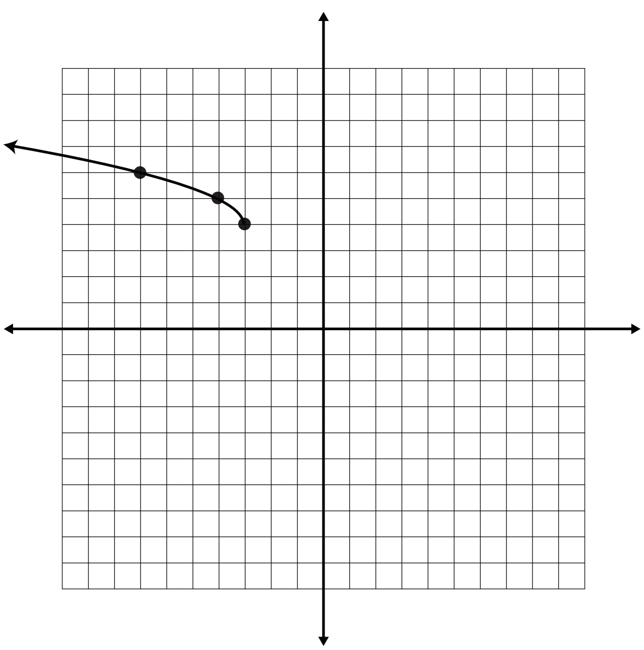
Example 6 For each graph, find the equation of the inverse.

Finding an Inverse From a Graph

a)



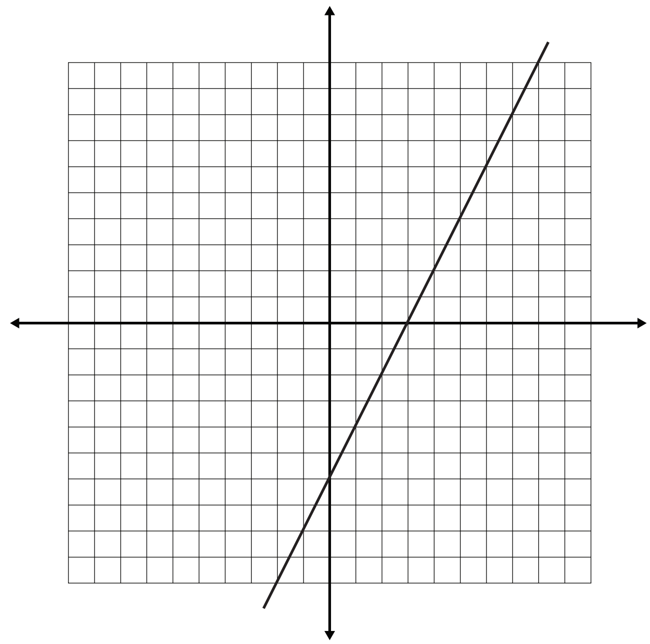
b)



Example 7 Answer the following questions.

Understanding Inverse Function Notation

a) If f(x) = 2x - 6, find the inverse function and determine the value of $f^{-1}(10)$.



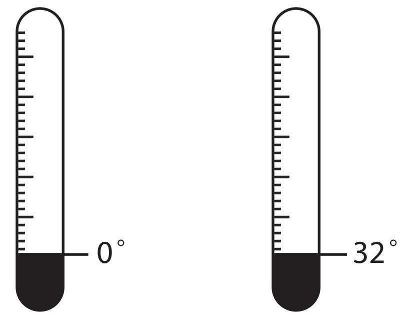
b) Given that f(x) has an inverse function $f^{-1}(x)$, is it true that if f(a) = b, then $f^{-1}(b) = a$?

c) If $f^{-1}(4) = 5$, determine f(5).

d) If $f^{-1}(k) = 18$, determine the value of *k*.

Example 8

In the Celsius temperature scale, the freezing point of water is set at 0 degrees. In the Fahrenheit temperature scale, 32 degrees is the freezing point of water. The formula to convert degrees Celsius to degrees Fahrenheit is: $F(C) = \frac{9}{5}C + 32$



Celsius Thermometer   Fahrenheit Thermometer

a) Determine the temperature in degrees Fahrenheit for 28 °C.

b) Derive a function, C(F), to convert degrees Fahrenheit to degrees Celsius. Does one need to understand the concept of an inverse to accomplish this?

c) Use the function C(F) from part (b) to determine the temperature in degrees Celsius for 100 °F.

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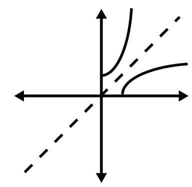
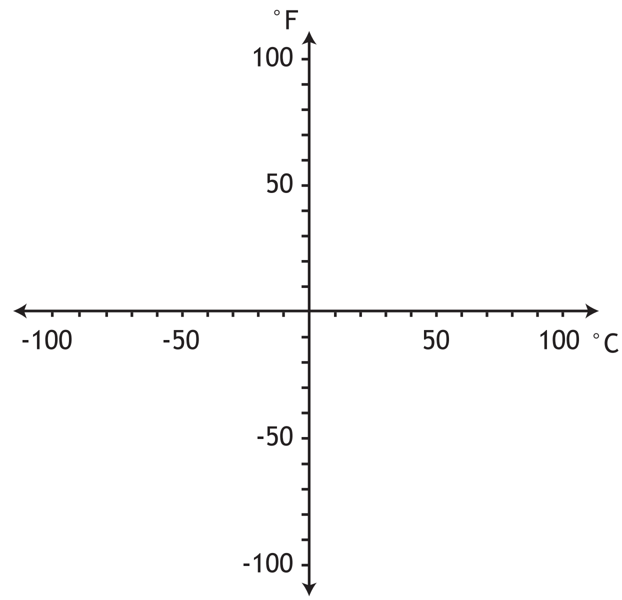


d) What difficulties arise when you try to graph F(C) and C(F) on the same grid?



e) Derive $F^{-1}(C)$. How does $F^{-1}(C)$ fix the graphing problem in part (d)?

f) Graph F(C) and $F^{-1}(C)$ using the graph above. What does the invariant point for these two graphs represent?

$(f + g)(x)$ $(f - g)(x)$

$(f \bullet g)(x)$ $\left(\frac{f}{g}\right)(x)$

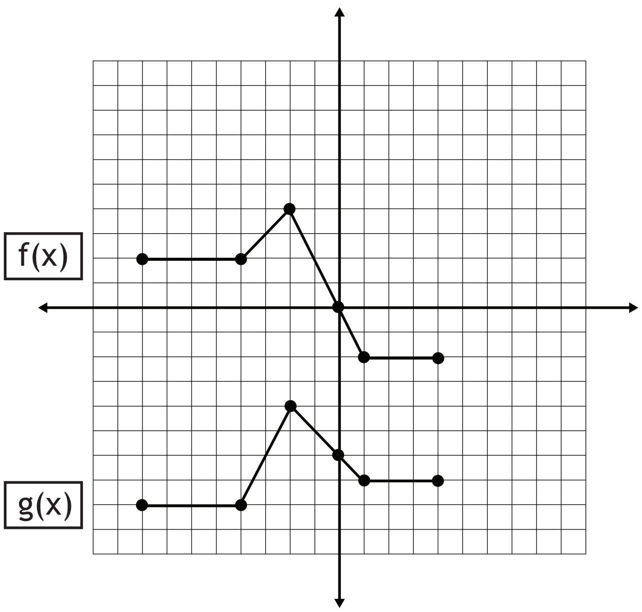
Transformations and Operations
LESSON FOUR - *Function Operations*
Lesson Notes

Example 1

Given the functions f(x) and g(x), complete the table of values for each operation and draw the graph. State the domain and range of the combined function.

Function Operations
(with a table of values)

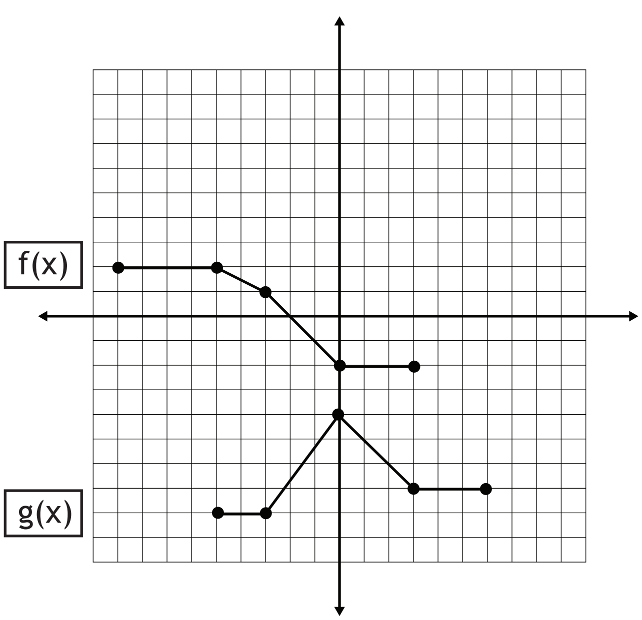
a) h(x) = (f + g)(x) *same as f(x) + g(x)*



x	*(f + g)(x)*
-8	
-4	
-2	
0	
1	
4	

Domain & Range:

b) h(x) = (f - g)(x) *same as f(x) - g(x)*



x	*(f - g)(x)*
-9	
-5	
-3	
0	
3	
6	

Domain & Range:

Set-Builder Notation

A ***set*** is simply a collection of numbers, such as {1, 4, 5}. We use ***set-builder notation*** to outline the rules governing members of a set.

{x | x ε R, x ≥ -1}

State the variable. List conditions on the variable.

In words: "The variable is x, such that x can be any real number with the condition that x ≥ -1".

As a shortcut, set-builder notation can be reduced to just the most important condition.

x ≥ -1

While this resource uses the shortcut for brevity, as set-builder notation is covered in previous courses, Math 30-1 students *are* expected to know how to read and write full set-builder notation.

Interval Notation

Math 30-1 students are expected to know that domain and range can be expressed using ***interval notation***.

() - Round Brackets: Exclude point from interval.
[] - Square Brackets: Include point in interval.
Infinity ∞ always gets a round bracket.

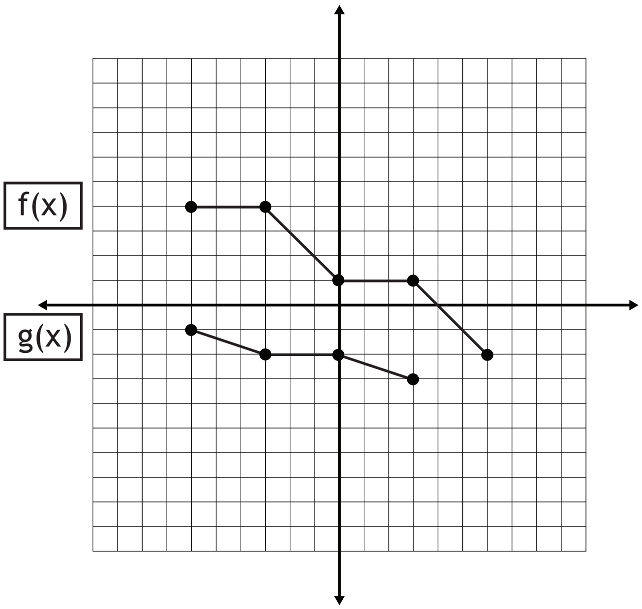
Examples: x ≥ -5 becomes [-5, ∞);
1 < x ≤ 4 becomes (1, 4];
x ε R becomes (-∞ , ∞);
-8 ≤ x < 2 or 5 ≤ x < 11
becomes [-8, 2) U [5, 11),
where U means "or", or ***union of sets***;
x ε R, x ≠ 2 becomes (-∞ , 2) U (2, ∞);
-1 ≤ x ≤ 3, x ≠ 0 becomes [-1 , 0) U (0, 3].

Transformations and Operations
LESSON FOUR - *Function Operations*
Lesson Notes

(f + g)(x) ***(f - g)(x)***
(f • g)(x) $\left(\frac{f}{g}\right)(x)$

Function Operations
(with a table of values)

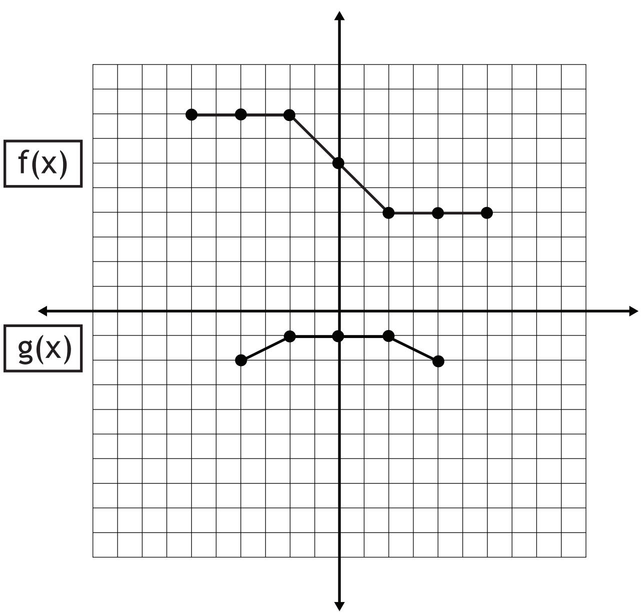
c) h(x) = (f • g)(x) *same as f(x) • g(x)*



x	*(f • g)(x)*
-6	
-3	
0	
3	
6	

Domain & Range:

d) $h(x) = \left(\frac{f}{g}\right)(x)$ *same as f(x) ÷ g(x)*



x	*(f ÷ g)(x)*
-6	
-4	
-2	
0	
2	
4	
6	

Domain & Range:

Transformations and Operations

LESSON FOUR - *Function Operations*

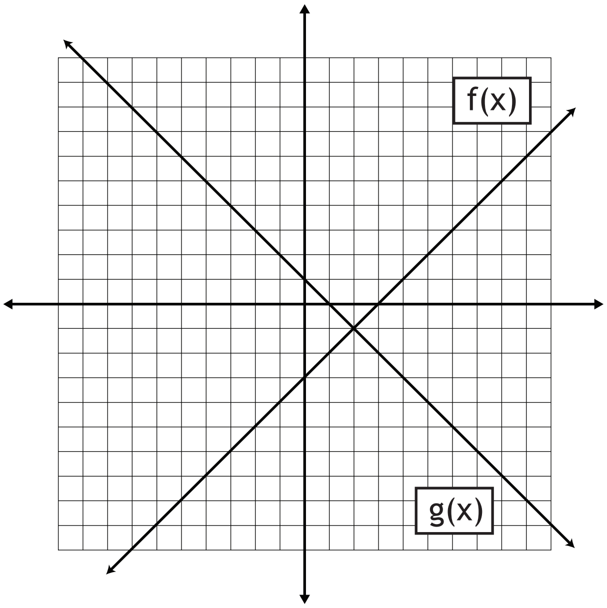
Lesson Notes

Example 2

Given the functions f(x) = x - 3 and g(x) = -x + 1, evaluate:

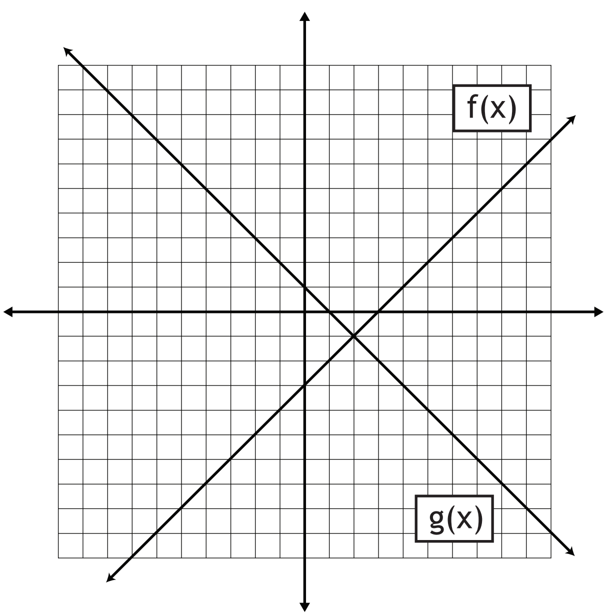
Function Operations
(graphically and algebraically)

a) (f + g)(-4) *same as f(-4) + g(-4)*



i) using the graph	ii) using h(x) = (f + g)(x)

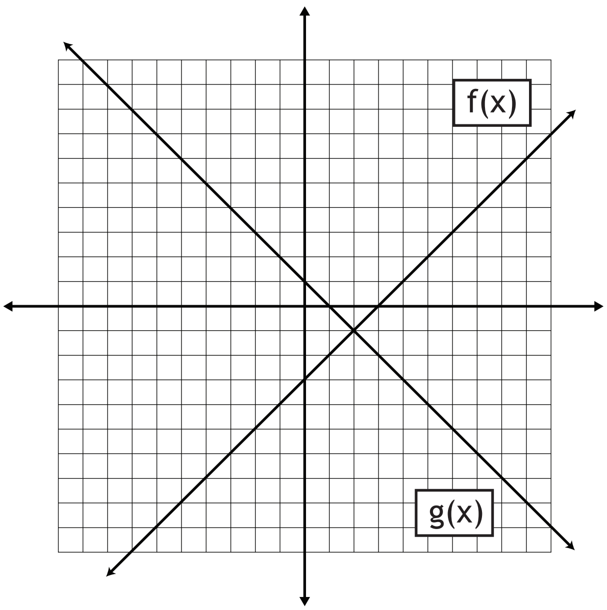
b) (f - g)(6) *same as f(6) - g(6)*



i) using the graph	ii) using h(x) = (f - g)(x)

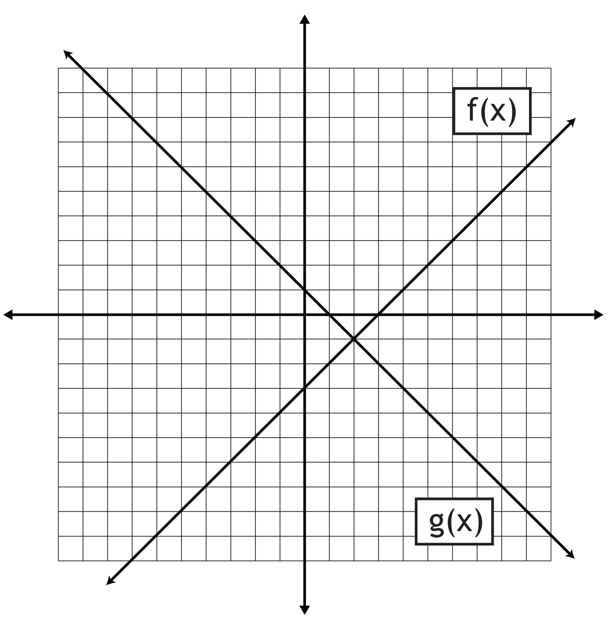
Function Operations
(graphically and algebraically)

c) (fg)(-1) *same as f(-1) • g(-1)*



i) using the graph	ii) using h(x) = (f • g)(x)

d) $\left(\frac{f}{g}\right)(5)$ *same as f(5) ÷ g(5)*



i) using the graph	ii) using h(x) = (f ÷ g)(x)

$(f + g)(x)$ $(f - g)(x)$

$(f \cdot g)(x)$ $\left(\frac{f}{g}\right)(x)$

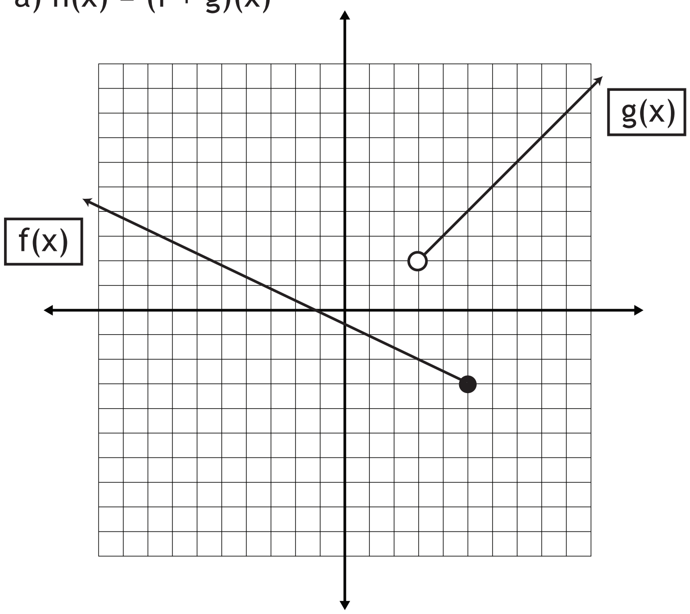
Transformations and Operations
LESSON FOUR - *Function Operations*
Lesson Notes

Example 3

Draw each combined function and state the domain and range.

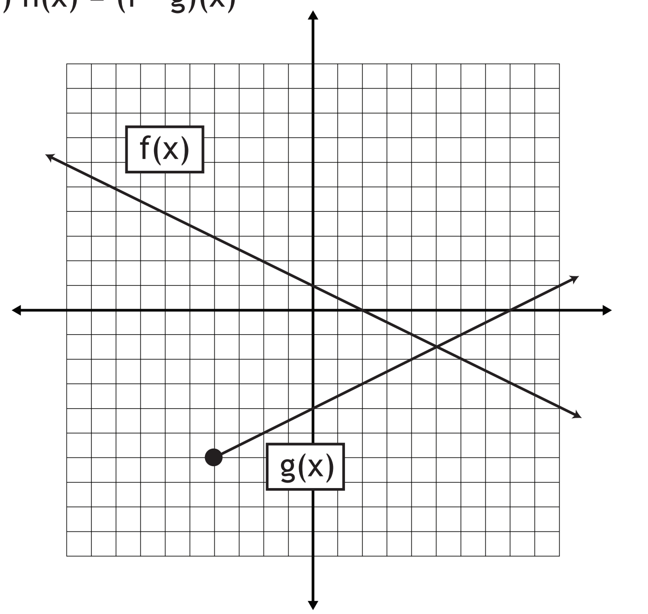
Combining Existing Graphs

a) h(x) = (f + g)(x)



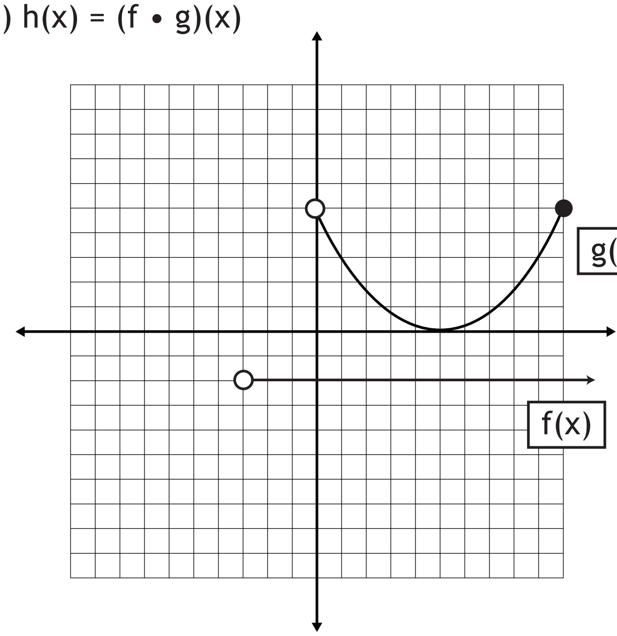
Domain & Range of h(x):

b) h(x) = (f - g)(x)



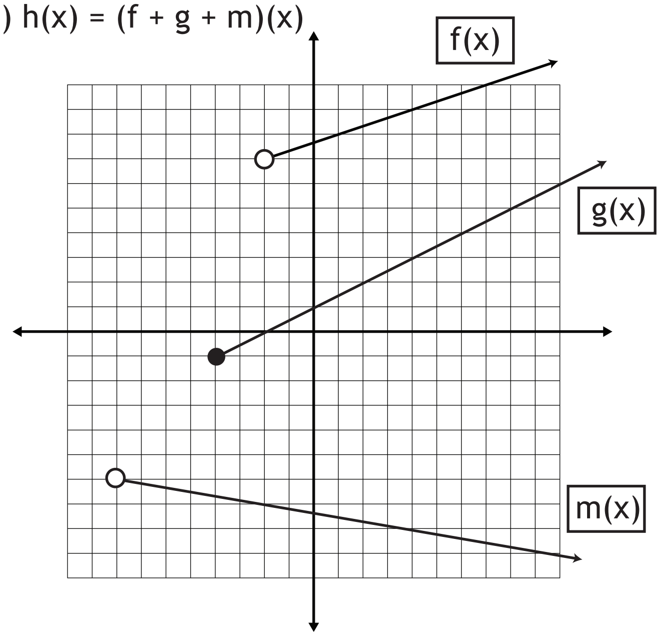
Domain & Range of h(x):

c) h(x) = (f • g)(x)



Domain & Range of h(x):

d) h(x) = (f + g + m)(x)



Domain & Range of h(x):

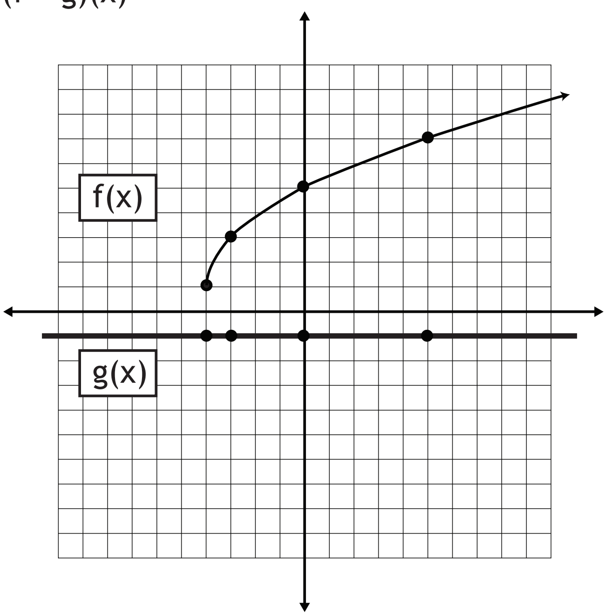
Transformations and Operations
LESSON FOUR - *Function Operations*
Lesson Notes

$(f + g)(x)$ $(f - g)(x)$
$(f \bullet g)(x)$ $\left(\frac{f}{g}\right)(x)$

Example 4 Given the functions $f(x) = 2\sqrt{x + 4} + 1$ and $g(x) = -1$, answer the following questions.

Function Operations
(with a radical function)

a) $(f + g)(x)$



i) Use a table of values to draw $(f + g)(x)$.

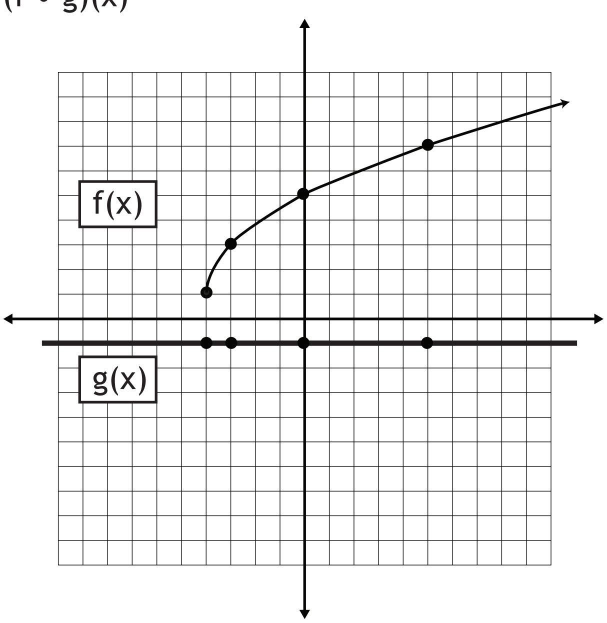
x	$(f + g)(x)$
-4	
-3	
0	
5	

ii) Derive $h(x) = (f + g)(x)$

iii) Domain & Range of h(x)

iv) Write a transformation equation that transforms the graph of f(x) to h(x).

b) $(f \bullet g)(x)$



i) Use a table of values to draw $(f \bullet g)(x)$.

x	$(f \bullet g)(x)$
-4	
-3	
0	
5	

ii) Derive $h(x) = (f \bullet g)(x)$

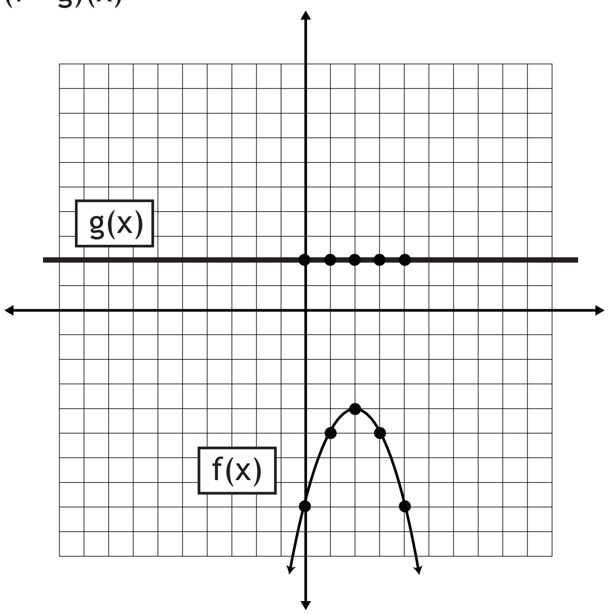
iii) Domain & Range of h(x)

iv) Write a transformation equation that transforms the graph of f(x) to h(x).

Example 5 Given the functions $f(x) = -(x - 2)^2 - 4$ and $g(x) = 2$, answer the following questions.

Function Operations
(with a quadratic function)

a) $(f - g)(x)$



i) Use a table of values to draw $(f - g)(x)$.

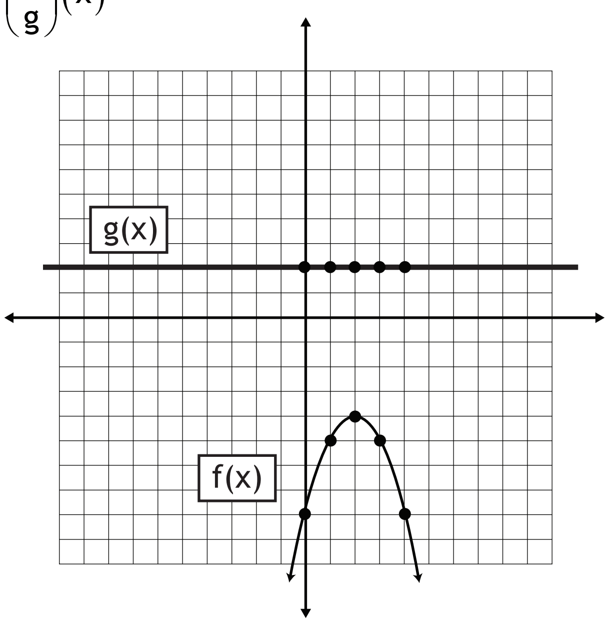
x	(f - g)(x)
0	
1	
2	
3	
4	

ii) Derive $h(x) = (f - g)(x)$

iii) Domain & Range of $h(x)$

iv) Write a transformation equation that transforms the graph of $f(x)$ to $h(x)$.

b) $\left(\frac{f}{g}\right)(x)$



i) Use a table of values to draw $(f \div g)(x)$.

x	(f ÷ g)(x)
0	
1	
2	
3	
4	

ii) Derive $h(x) = (f \div g)(x)$

iii) Domain & Range of $h(x)$

iv) Write a transformation equation that transforms the graph of $f(x)$ to $h(x)$.

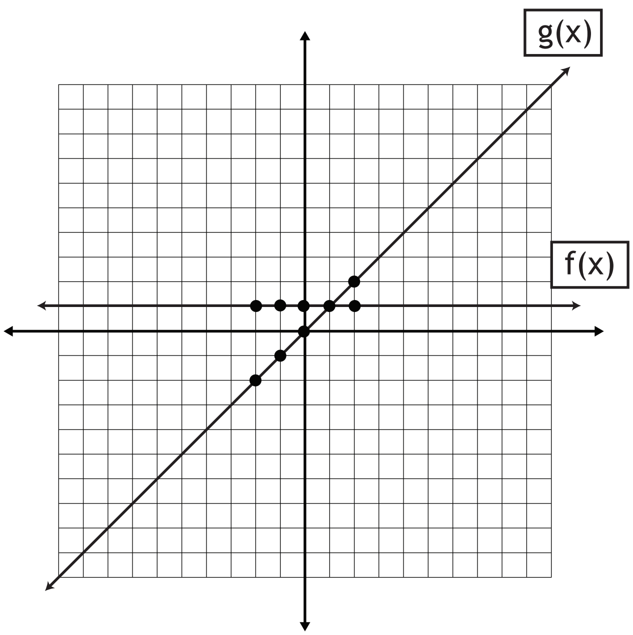
$(f + g)(x)$　**$(f - g)(x)$**

$(f \cdot g)(x)$　**$\left(\frac{f}{g}\right)(x)$**

Example 6 Draw the graph of $h(x) = \left(\frac{f}{g}\right)(x)$. Derive h(x) and state the domain and range.

Function Operations
(with a rational function)

a) f(x) = 1 and g(x) = x



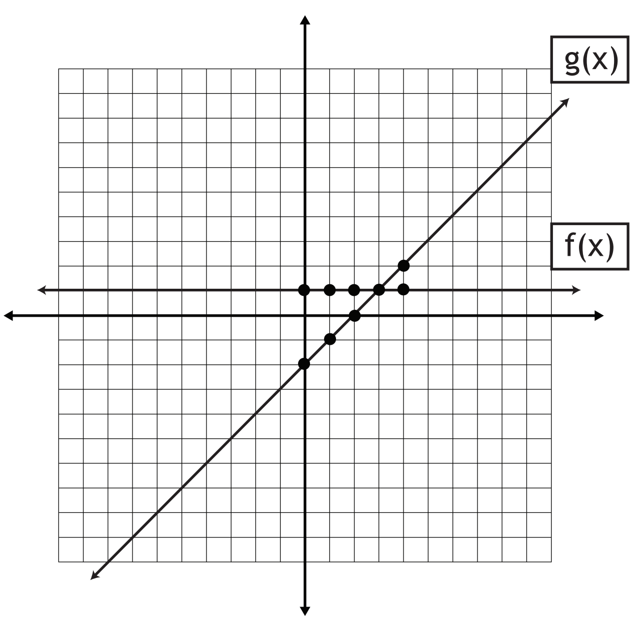
i) Use a table of values to draw (f ÷ g)(x).

x	(f ÷ g)(x)
-2	
-1	
0	
1	
2	

ii) Derive h(x) = (f ÷ g)(x)

iii) Domain & Range of h(x)

b) f(x) = 1 and g(x) = x - 2



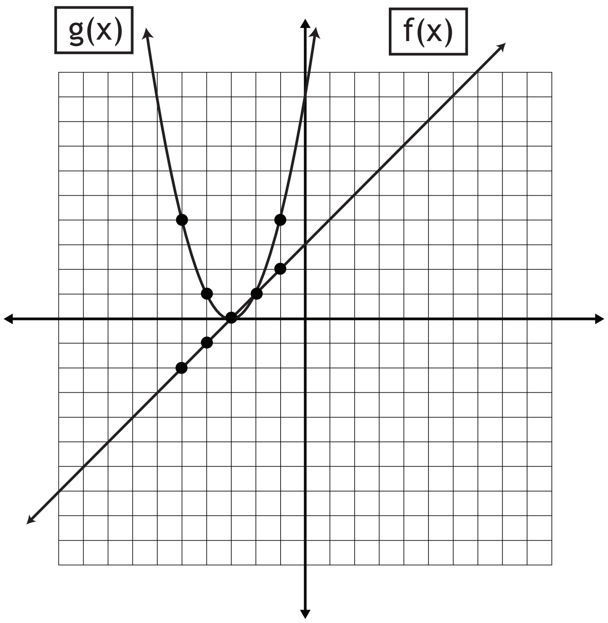
i) Use a table of values to draw (f ÷ g)(x).

x	(f ÷ g)(x)
0	
1	
2	
3	
4	

ii) Derive h(x) = (f ÷ g)(x)

iii) Domain & Range of h(x)

c) $f(x) = x + 3$ and $g(x) = x^2 + 6x + 9$



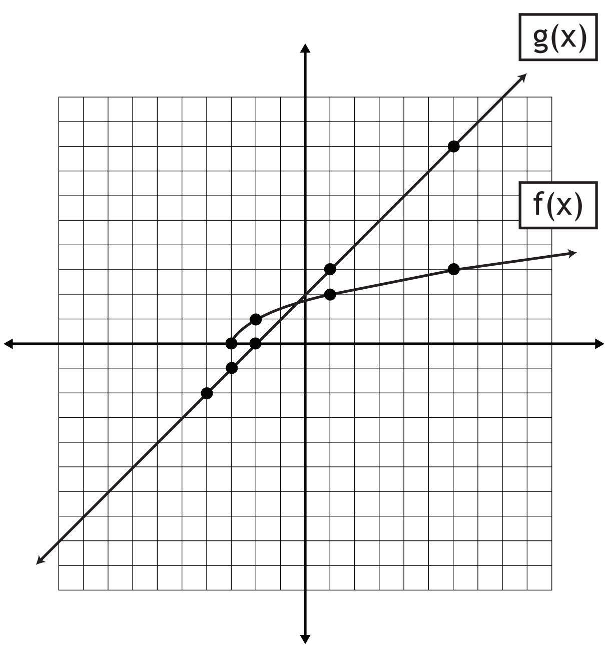
i) Use a table of values to draw $(f \div g)(x)$.

x	$(f \div g)(x)$
-5	
-4	
-3	
-2	
-1	

ii) Derive $h(x) = (f \div g)(x)$

iii) Domain & Range of h(x)

d) $f(x) = \sqrt{x + 3}$ and $g(x) = x + 2$



i) Use a table of values to draw $(f \div g)(x)$.

x	$(f \div g)(x)$
-4	
-3	
-2	
1	
6	

ii) Derive $h(x) = (f \div g)(x)$

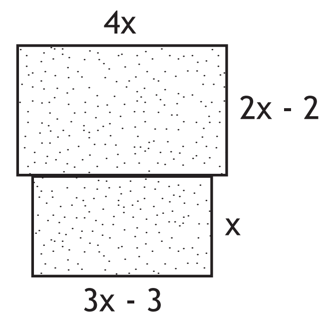
iii) Domain & Range of h(x)

(f + g)(x) ***(f - g)(x)***

(f • g)(x) $\left(\frac{f}{g}\right)(x)$

Example 7

Two rectangular lots are adjacent to each other, as shown in the diagram.



a) Write a function, $A_L(x)$, for the area of the large lot.

b) Write a function, $A_S(x)$, for the area of the small lot.

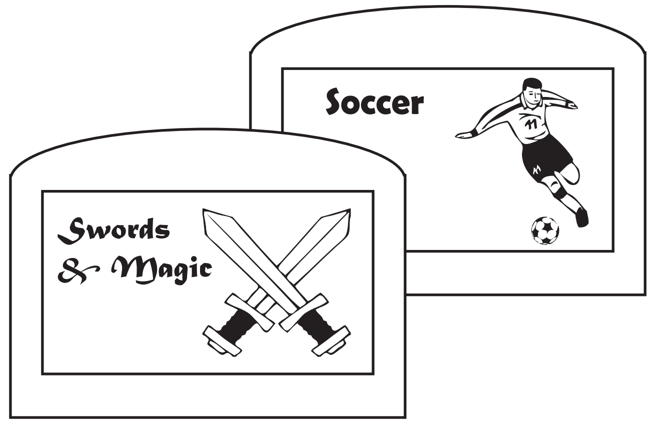
c) If the large rectangular lot is 10 m^2 larger than the small lot, use a function operation to solve for x.

d) Using a function operation, determine the total area of both lots.

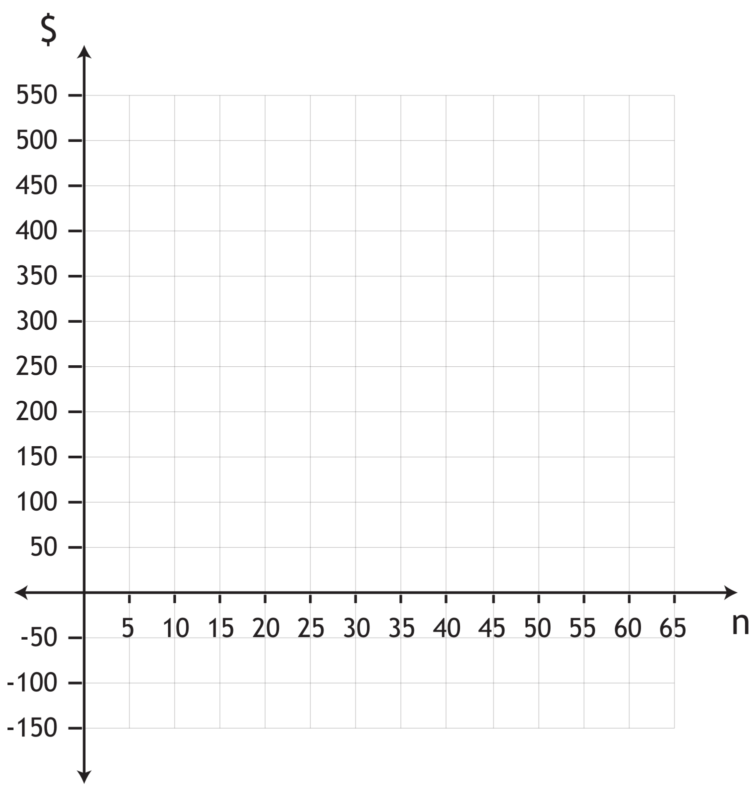
e) Using a function operation, determine how many times bigger the large lot is than the small lot.

Example 8

Greg wants to to rent a stand at a flea market to sell old video game cartridges. He plans to acquire games for $4 each from an online auction site, then sell them for $12 each. The cost of renting the stand is $160 for the day.



a) Using function operations, derive functions for revenue *R(n)*, expenses *E(n)*, and profit *P(n)*. Graph each function.



b) What is Greg's profit if he sells 52 games?

c) How many games must Greg sell to break even?

Transformations and Operations
LESSON FOUR - *Function Operations*
Lesson Notes

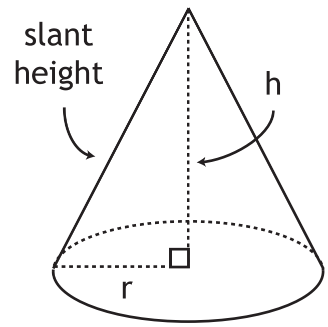
$(f + g)(x)$ $(f - g)(x)$

$(f \cdot g)(x)$ $\left(\frac{f}{g}\right)(x)$

Example 9

The surface area and volume of a right cone are:

$$SA = \pi r^2 + \pi rs$$
$$V = \frac{1}{3}\pi r^2 h$$



where r is the radius of the circular base, h is the height of the apex, and s is the slant height of the side of the cone.

A particular cone has a height that is $\sqrt{3}$ times larger than the radius.

a) Can we write the surface area and volume formulae as single-variable functions?

b) Express the apex height in terms of r.

c) Express the slant height in terms of r.

d) Rewrite both the surface area and volume formulae so they are single-variable functions of r.

e) Use a function operation to determine the surface area to volume ratio of the cone.

f) If the radius of the base of the cone is 6 m, find the exact value of the surface area to volume ratio.

$f \circ g = f(g(x))$

Transformations and Operations
LESSON FIVE - *Function Composition*
Lesson Notes

Example 1 Given the functions $f(x) = x - 3$ and $g(x) = x^2$:

Function Composition
(tables of values and two function machines)

a) Complete the table of values for $(f \circ g)(x)$. *same as f(g(x))*

x	g(x)	f(g(x))
-3		
-2		
-1		
0		
1		
2		
3		

b) Complete the table of values for $(g \circ f)(x)$. *same as g(f(x))*

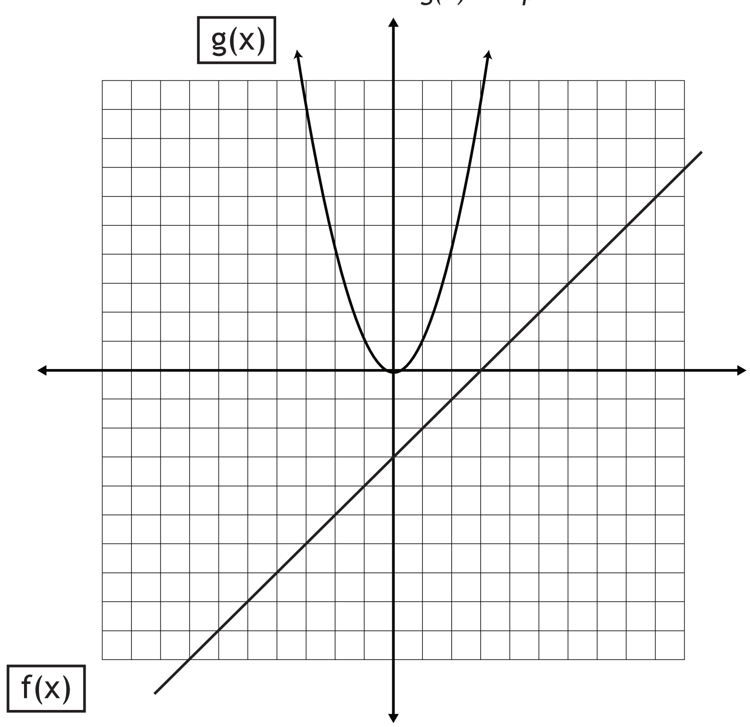
x	f(x)	g(f(x))
0		
1		
2		
3		

c) Does order matter when performing a composition?

d) Derive $m(x) = (f \circ g)(x)$.

e) Derive $n(x) = (g \circ f)(x)$.

f) Draw m(x) and n(x). *The graphs of f(x) and g(x) are provided.*



Transformations and Operations
LESSON FIVE - *Function Composition*
Lesson Notes

$$f \circ g = f(g(x))$$

Example 2

Given the functions $f(x) = x^2 - 3$ and $g(x) = 2x$, evaluate each of the following:

Function Composition
(numeric solution)

a) $m(3) = (f \circ g)(3)$

b) $n(1) = (g \circ f)(1)$

c) $p(2) = (f \circ f)(2)$

d) $q(-4) = (g \circ g)(-4)$

$f \circ g = f(g(x))$

Example 3

Given the functions $f(x) = x^2 - 3$ and $g(x) = 2x$ *(these are the same functions found in Example 2)*, find each composite function.

Function Composition
(algebraic solution)

a) $m(x) = (f \circ g)(x)$

b) $n(x) = (g \circ f)(x)$

c) $p(x) = (f \circ f)(x)$

d) $q(x) = (g \circ g)(x)$

e) Using the composite functions derived in parts (a - d), evaluate m(3), n(1), p(2), and q(-4). Do the results match the answers in Example 2?

Transformations and Operations
LESSON FIVE - *Function Composition*
Lesson Notes

$$f \circ g = f(g(x))$$

Example 4

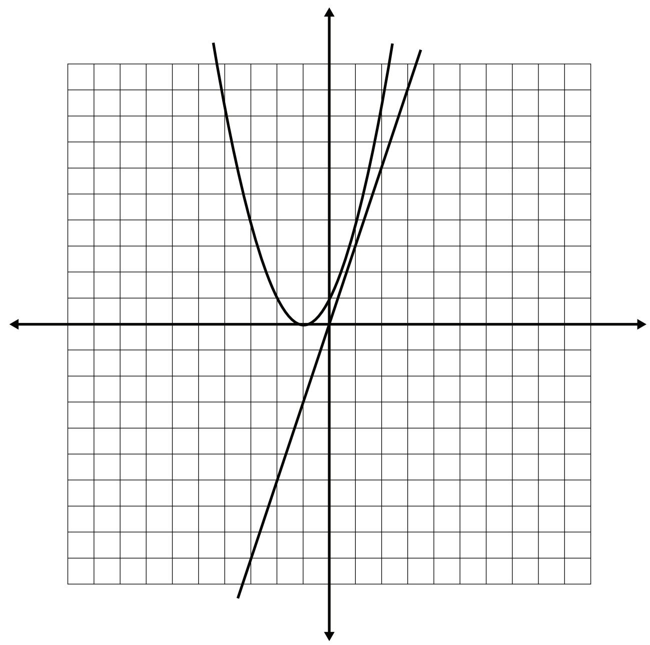
Given the functions f(x) and g(x), find each composite function. Make note of any transformations as you complete your work.

Function Composition and Transformations

$f(x) = (x + 1)^2 \qquad g(x) = 3x$

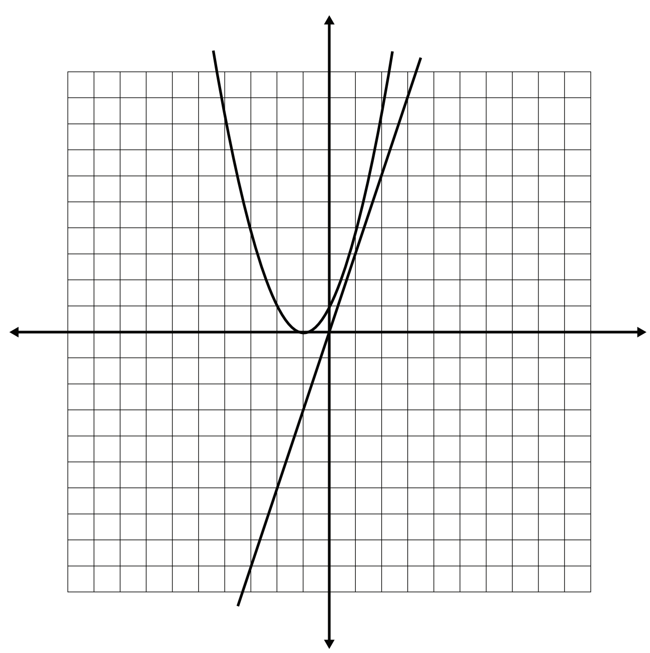
a) $m(x) = (f \circ g)(x)$

Transfomation:



b) $n(x) = (g \circ f)(x)$

Transfomation:



$f \circ g = f(g(x))$

Example 5

Given the functions f(x) and g(x), find the composite function m(x) = (f ∘ g)(x) and state the domain.

Domain of Composite Functions

a) $f(x) = \sqrt{x - 3}$
$g(x) = x - 5$

b) $f(x) = \sqrt{x - 3}$
$g(x) = x + 1$

$$f \circ g = f(g(x))$$

Example 6

Given the functions f(x), g(x), m(x), and n(x), find each composite function and state the domain.

Function Composition
(three functions)

$f(x) = \sqrt{x}$ $\quad g(x) = \frac{1}{x}$ $\quad m(x) = |x|$ $\quad n(x) = x + 2$

a) $h(x) = [g \circ m \circ n](x)$

b) $h(x) = [n \circ f \circ n](x)$

Example 7

Given the functions f(x), g(x), m(x), and n(x), find each composite function and state the domain.

Function Composition
(with additional operations)

$f(x) = \sqrt{x}$ $\quad g(x) = \frac{1}{x}$ $\quad m(x) = |x|$ $\quad n(x) = x + 2$

a) $h(x) = [(gg) \circ n](x)$

b) $h(x) = [f \circ (n + n)](x)$

Transformations and Operations
LESSON FIVE - *Function Composition*
Lesson Notes

$$f \circ g = f(g(x))$$

Example 8

Given the composite function $h(x) = (f \circ g)(x)$, find the component functions, $f(x)$ and $g(x)$. *(More than one answer is possible)*

Components of a Composite Function

a) $h(x) = 2x + 2$

b) $h(x) = \dfrac{1}{x^2 - 1}$

c) $h(x) = (x + 1)^2 - 5(x + 1) + 1$

d) $h(x) = x^2 + 4x + 4$

e) $h(x) = 2\sqrt{\dfrac{1}{x}}$

f) $h(x) = |x|$

Example 9

Two functions are inverses if $(f^{-1} \circ f)(x) = x$. Determine if each pair of functions are inverses of each other.

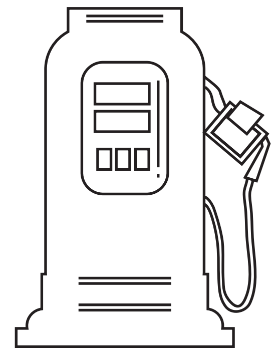
Composite Functions and Inverses

a) $f(x) = 3x - 2$ and $f^{-1}(x) = \frac{1}{3}x + \frac{2}{3}$

b) $f(x) = x - 1$ and $f^{-1}(x) = 1 - x$

$f \circ g = f(g(x))$

Example 10



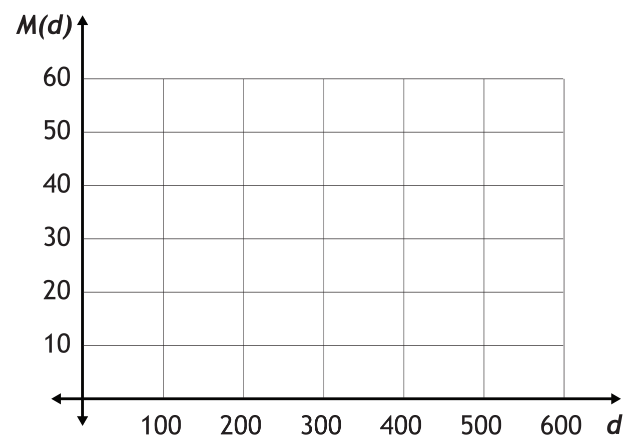
The price of 1 L of gasoline is $1.05. On a level road, Darlene's car uses 0.08 L of fuel for every kilometre driven.

a) If Darlene drives 50 km, how much did the gas cost to fuel the trip? How many steps does it take to solve this problem *(without composition)*?

b) Write a function, V(d), for the volume of gas consumed as a function of the distance driven.

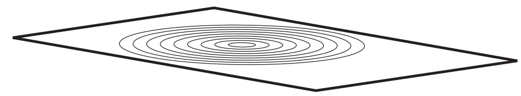
c) Write a function, M(V), for the cost of the trip as a function of gas volume.

d) Using function composition, combine the functions from parts b & c into a single function, M(d), where M is the money required for the trip. Draw the graph.



e) Solve the problem from part (a) again, but this time use the function derived in part (d). How many steps does the calculation take now?

Example 11



A pebble dropped in a lake creates a circular wave that travels outward at a speed of 30 cm/s.

a) Use function composition to derive a function, A(t), that expresses the area of the circular wave as a function of time.

b) What is the area of the circular wave after 3 seconds?

c) How long does it take for the area enclosed by the circular wave to be 44100π cm^2?
What is the radius of the wave?

$f \circ g = f(g(x))$

Example 12

$CAD
$USD
€
¥
£

The exchange rates of several currencies on a particular day are listed below:

American Dollars = 1.03 × Canadian Dollars
Euros = 0.77 × American Dollars
Japanese Yen = 101.36 × Euros
British Pounds = 0.0083 × Japanese Yen

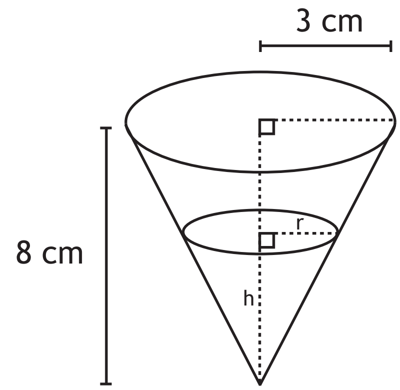
a) Write a function, a(c), that converts Canadian dollars to American dollars.

b) Write a function, j(a), that converts American Dollars to Japanese Yen.

c) Write a function, b(a), that converts American Dollars to British Pounds.

d) Write a function, b(c), that converts Canadian Dollars to British Pounds.

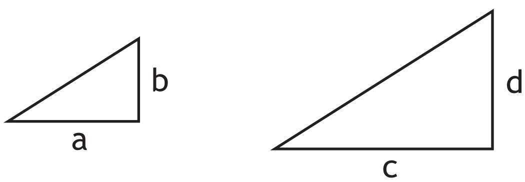
Example 13



A drinking cup from a water fountain has the shape of an inverted cone. The cup has a height of 8 cm, and a radius of 3 cm. The water in the cup also has the shape of an inverted cone, with a radius of r and a height of h.

The diagram of the drinking cup shows two right triangles: a large triangle for the entire height of the cup, and a smaller triangle for the water in the cup. The two triangles have identical angles, so they can be classified as similar triangles.

Reminder: In similar triangles, the ratios of corresponding sides are equal.



$$\frac{d}{b} = \frac{c}{a}$$

Reminder: The volume of a cone is:

$$V = \frac{1}{3}\pi r^2 h$$

a) Use similar triangle ratios to express r as a function of h.

b) Derive the composite function, $V_{water}(h) = (V_{cone} \circ r)(h)$, for the volume of the water in the cone.

c) If the volume of water in the cone is 3π cm^3, determine the height of the water.

$f \circ g = f(g(x))$

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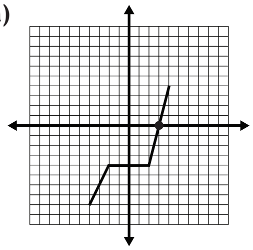
Answer Key

Transformations and Operations Lesson One: Basic Transformations

On this page,
● = invariant point

Example 1: a)

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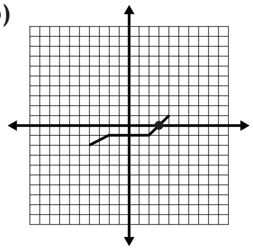
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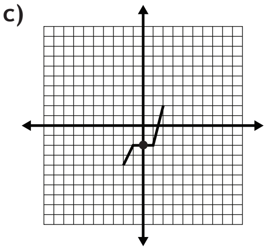
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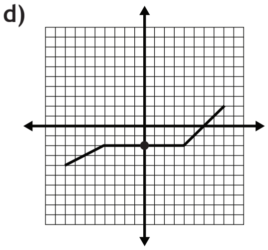

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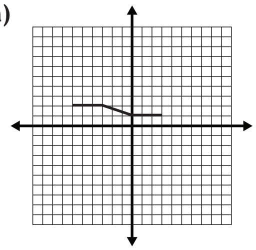
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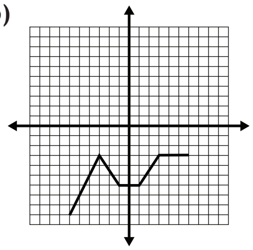
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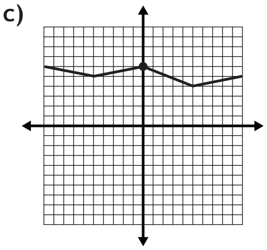
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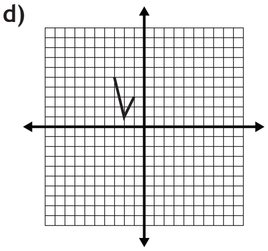
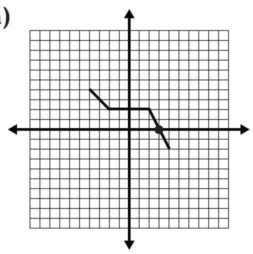


Example 3: a)

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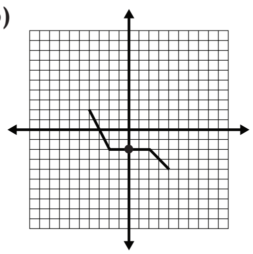
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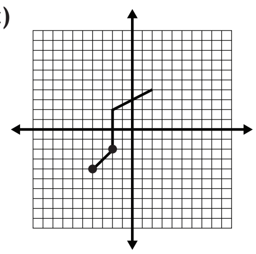
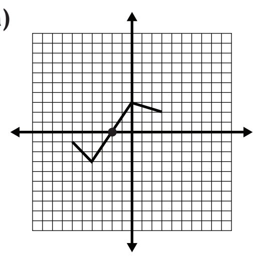


Example 4: a)

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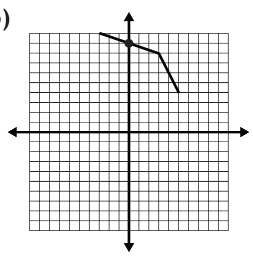
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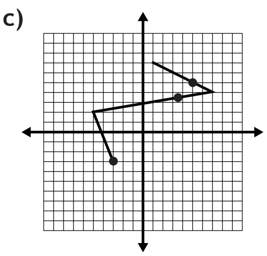
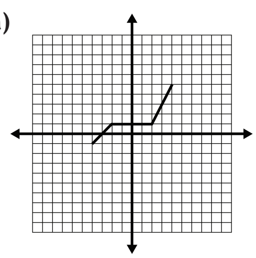


Example 5: a)

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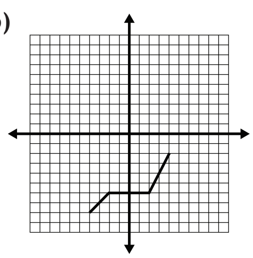
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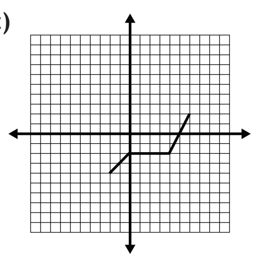
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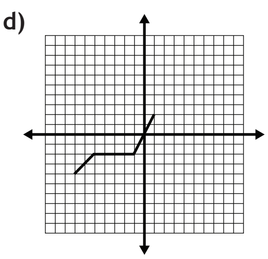
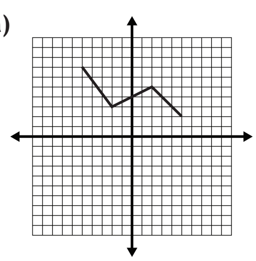


Example 6: a)

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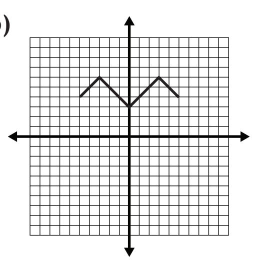
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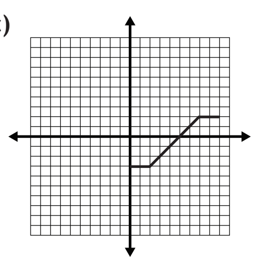
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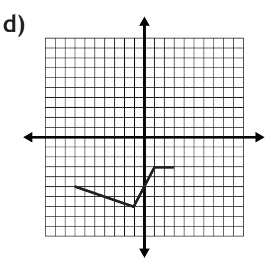
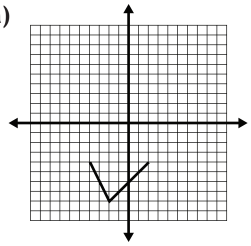


Answer Key

Example 7:

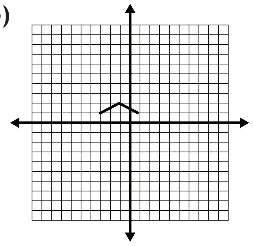
a)



$y = f(2x)$

$(x, y) \to \left(\frac{1}{2}x, y\right)$

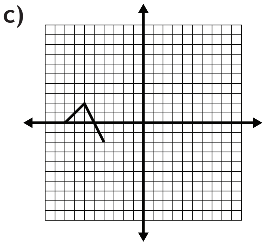
b)



$y = f(x+6)$

$(x, y) \to (x-6, y)$

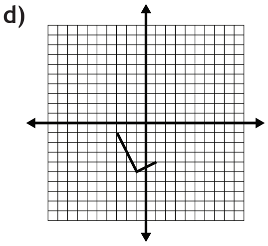
c)



$y = f(x) - 4$

$(x, y) \to (x, y-4)$

d)



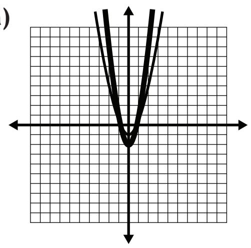
$y = -f(x)$

$(x, y) \to (x, -y)$

Example 8: **a)** $y = f(x) - 4$ **b)** $y = f(3x)$ **c)** $y = f\left(\frac{1}{2}x\right)$ **d)** $y = -f(x)$

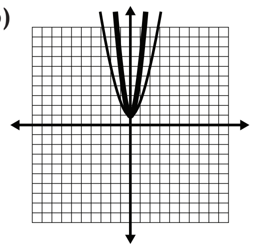
Example 9:

a)



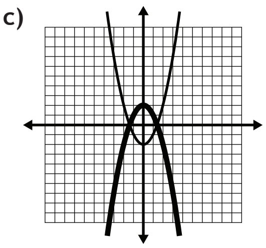
$y = 2x^2 - 2$

b)



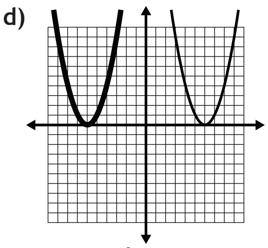
$y = 4x^2 + 1$

c)



$y = -x^2 + 2$

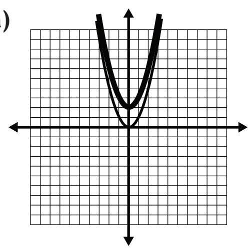
d)



$y = (-x-6)^2$

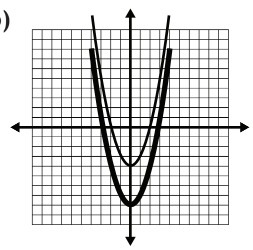
Example 10:

a)



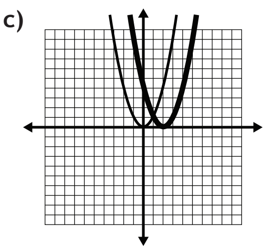
$y = x^2 + 2$

b)



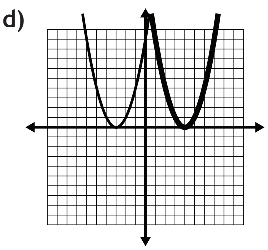
$y = x^2 - 8$

c)



$y = (x-2)^2$

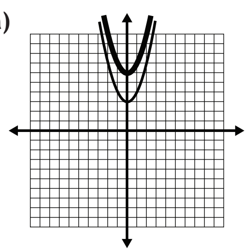
d)



$y = (x-4)^2$

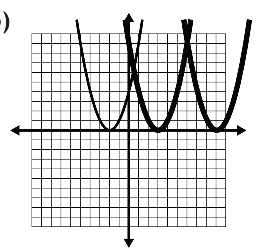
Example 11:

a)



$y = f(x) + 3$

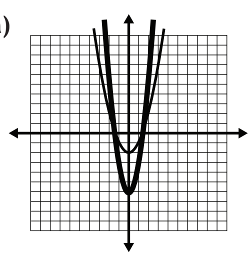
b)



$y = f(x-5)$ or $y = f(x-11)$

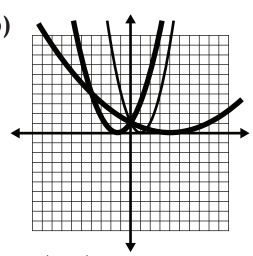
Example 12:

a)



$y = 3f(x)$

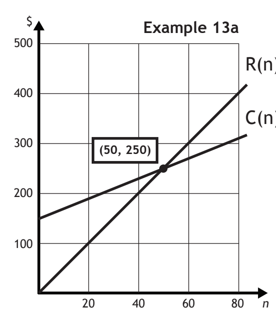
b)

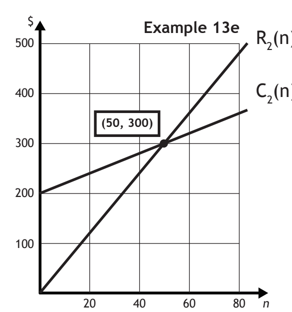


$y = f\left(\frac{1}{4}x\right)$ or $y = f\left(-\frac{3}{4}x\right)$

Example 13:

a) R(n) = 5n
C(n) = 2n + 150
b) 50 loaves
c) $C_2(n)$ = 2n + 200
d) $R_2(n)$ = 6n
e) 50 loaves





Example 14:

a) $h(d-2) = -\frac{1}{9}(d-6)^2 + 4$

b) 12 metres

Answer Key

Transformations and Operations Lesson Two: Combined Transformations

Example 1: **a)** ***a*** is the vertical stretch factor. ***b*** is the reciprocal of the horizontal stretch factor. ***h*** is the horizontal displacement. ***k*** is the vertical displacement.

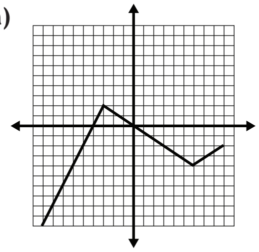
b) i. V.S. 1/3
H.S. 1/5

ii. V.S. 2
H.S. 4

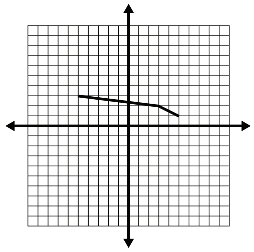
iii. V.S. 1/2
H.S. 3
Reflection about x-axis

iv. V.S. 3
H.S. 1/2
Reflection about x-axis
Reflection about y-axis

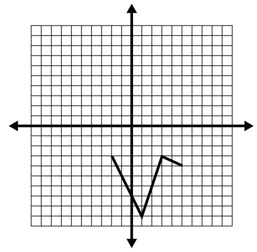
Example 2: **a)**



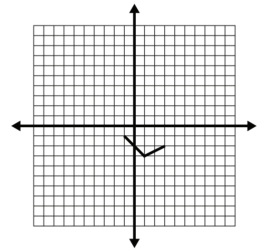
b)



c)



d)



Example 3: **a)** H.T. 3 left

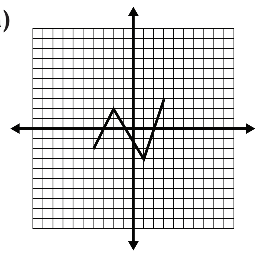
b) i. H.T. 1 right
V.T. 3 up

ii. H.T. 2 left
V.T. 4 down

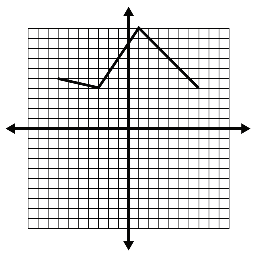
iii. H.T. 2 right
V.T. 3 down

iv. H.T. 7 left
V.T. 5 up

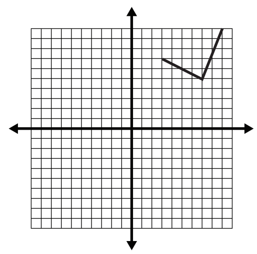
Example 4: **a)**



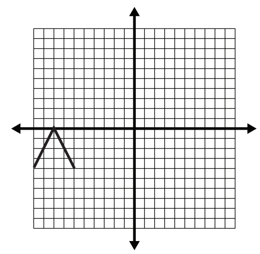
b)



c)



d)



Example 5: **a)** Stretches and reflections should be applied first, in any order. Translations should be applied last, in any order.

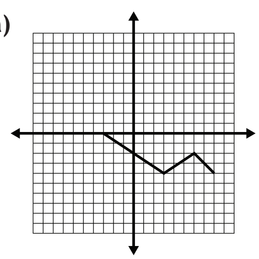
b) i. V.S. 2
H.T. 3 left
V.T. 1 up

ii. H.S. 3
Reflection about x-axis
V.T. 4 down

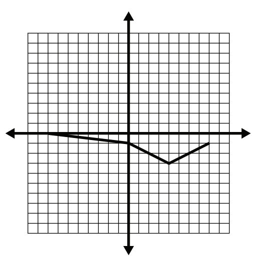
iii. V.S. 1/2
Reflection about y-axis
H.T. 2 left; V.T. 3 down

iv. V.S. 3; H.S. 1/4
Reflection about x-axis
Reflection about y-axis
H.T. 1 right; V.T. 2 up

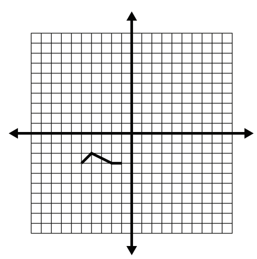
Example 6: **a)**



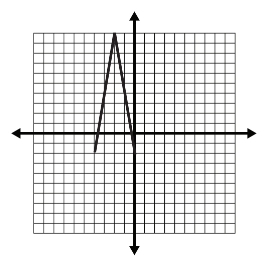
b)



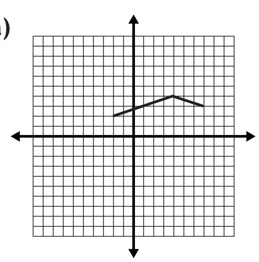
c)



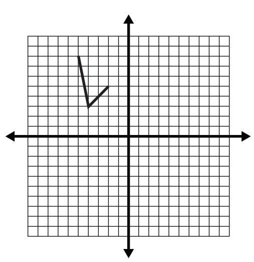
d)



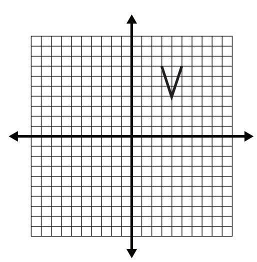
Example 7: **a)**



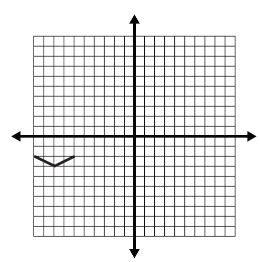
b)



c)

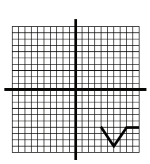


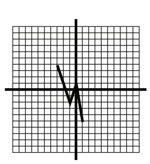
d)



Example 8:
a) (1, 0)
b) (3, 6)
c) m = 8 and n = 1

Example 9:
a) $y = -3f(x - 2)$
b) $y = -f[3(x + 2)]$





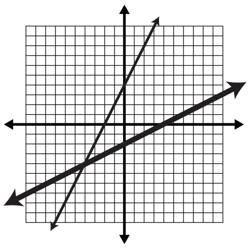
Example 10:
Axis-Independence
Apply all the vertical transformations together and apply all the horizontal transformations together, in either order.

Example 11:
a) H.T. 8 right; V.T. 7 up
b) Reflection about x-axis; H.T. 4 left; V.T. 6 down
c) H.S. 2; H.T. 3 left; V.T. 7 up
d) H.S. 1/2; Reflection about x & y-axis; H.T. 5 right; V.T. 7 down.
e) The spaceship is not a function, and it must be translated in a specific order to avoid the asteroids.

Answer Key

Transformations and Operations Lesson Three: Inverses

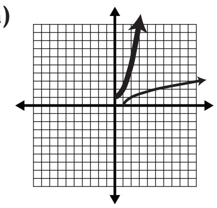
Example 1: **a)** Line of Symmetry: y = x



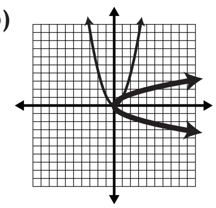
b) $f^{-1}(x) = \frac{1}{2}x - 2$

Example 2:

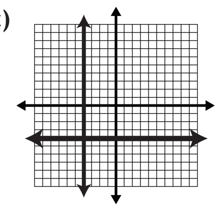
a)



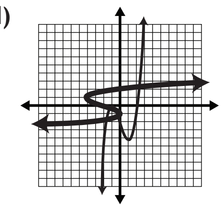
b)



c)

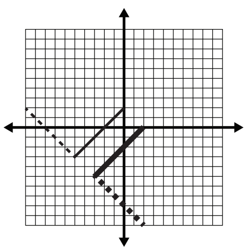


d)



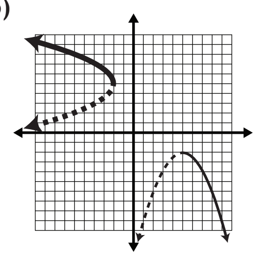
Example 3:

a)



Restrict the domain of the original function to -10 ≤ x ≤ -5 or -5 ≤ x ≤ 0

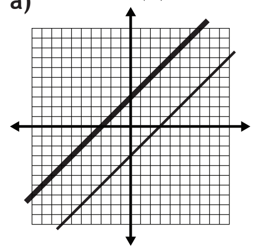
b)



Restrict the domain of the original function to x ≤ 5 or x ≥ 5.

Example 4:

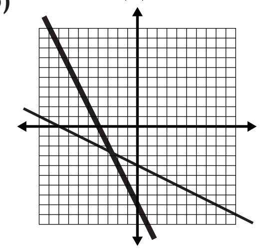
a) $f^{-1}(x) = x + 3$



Original:	Inverse:
D: x ε R	D: x ε R
R: y ε R	R: y ε R

The inverse is a function.

b) $f^{-1}(x) = -2x - 8$

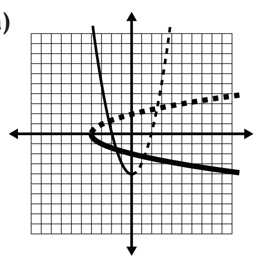


Original:	Inverse:
D: x ε R	D: x ε R
R: y ε R	R: y ε R

The inverse is a function.

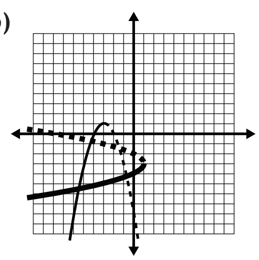
Example 5:

a)



Restrict the domain of the original function to x ≤ 0 or x ≥ 0

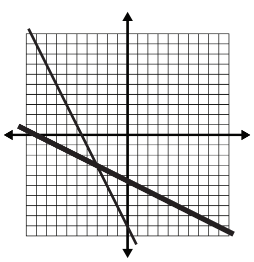
b)



Restrict the domain of the original function to x ≤ -3 or x ≥ -3.

Example 6:

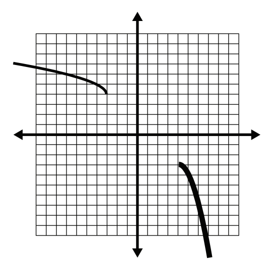
a)



$f^{-1}(x) = -\frac{1}{2}x - \frac{9}{2}$

D: x ε R

b)



$f^{-1}(x) = -(x - 4)^2 - 3$

D: x ≥ 4

Example 7:

a) (10, 8)

b) True.
$f^{-1}(b) = a$

c) f(5) = 4

d) k = 30

Example 8:

a) 28 °C is equivalent to 82.4 °F

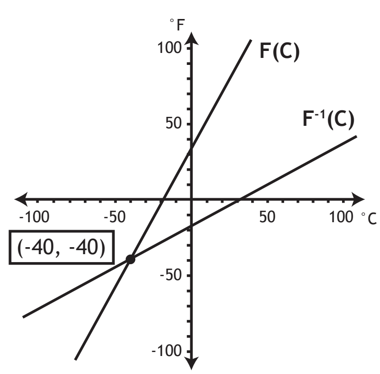
b) $C(F) = \frac{5}{9}F - \frac{160}{9}$

c) 100 °F is equivalent to 37.8 °C

d) C(F) can't be graphed since its dependent variable is C, but the dependent variable on the graph's y-axis is F. This is a mismatch.

e) $F^{-1}(C) = \frac{5}{9}C - \frac{160}{9}$

f) The invariant point occurs when the temperature in degrees Fahrenheit is equal to the temperature in degrees Celsius. -40 °F is equal to -40 °C.

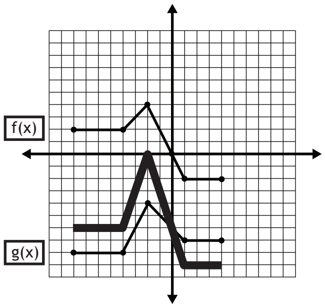


Answer Key

Transformations and Operations Lesson Four: Function Operations

Example 1: **a)**

x	(f + g)(x)
-8	-6
-4	-6
-2	0
0	-6
1	-9
4	-9

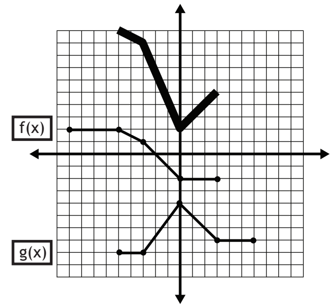


Domain:
$-8 \le x \le 4$
or [-8, 4]

Range:
$-9 \le y \le 0$
or [-9, 0]

b)

x	(f - g)(x)
-9	DNE
-5	10
-3	9
0	2
3	5
6	DNE

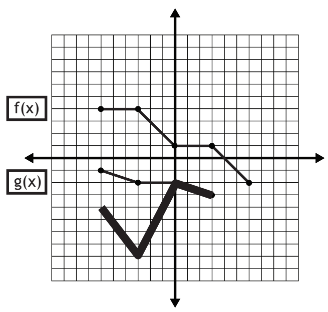


Domain:
$-5 \le x \le 3$;
or [-5, 3]

Range:
$2 \le y \le 10$
or [2, 10]

c)

x	(f • g)(x)
-6	-4
-3	-8
0	-2
3	-3
6	DNE

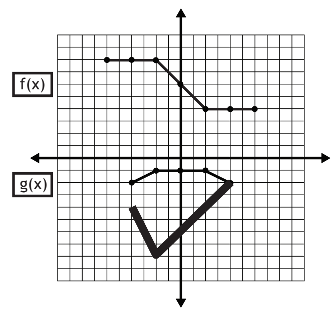


Domain:
$-6 \le x \le 3$
or [-6, 3]

Range:
$-8 \le y \le -2$
or [-8, -2]

d)

x	(f ÷ g)(x)
-6	DNE
-4	-4
-2	-8
0	-6
2	-4
4	-2
6	DNE



Domain:
$-4 \le x \le 4$
or [-4, 4]

Range:
$-8 \le y \le -2$
or [-8, -2]

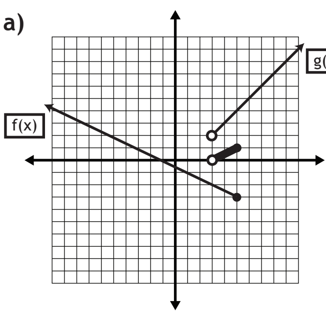
Example 2:

a) i. (f + g)(-4) = -2   ii. h(x) = -2; h(-4) = -2
b) i. (f – g)(6) = 8   ii. h(x) = 2x – 4; h(6) = 8
c) i. (fg)(-1) = -8   ii. h(x) = $-x^2 + 4x - 3$; h(-1) = -8
d) i. (f/g)(5) = -0.5   ii. h(x) = (x - 3)/(-x + 1); h(5) = -0.5

> **Reminder:** Math 30-1 students are expected to know that domain and range can be expressed using ***interval notation***.

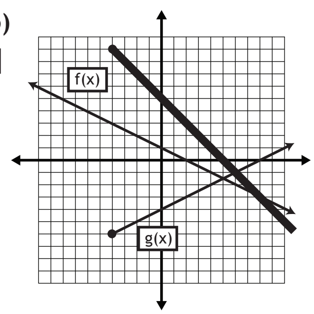
Example 3:

a)



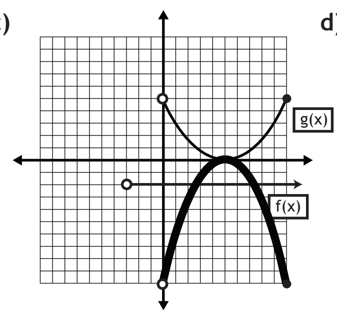
Domain: $3 < x \le 5$ or (3, 5]
Range: $0 < y \le 1$ or (0, 1]

b)



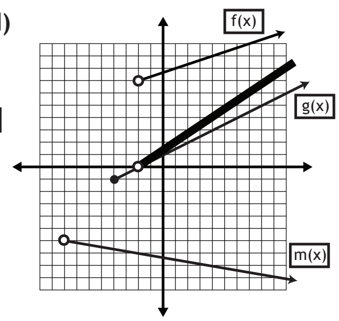
Domain: $x \ge -4$ or [-4, ∞)
Range: $y \le 9$ or (-∞, 9]

c)



Domain: $0 < x \le 10$ or (0, 10]
Range: $-10 \le y \le 0$ or [-10, 0]

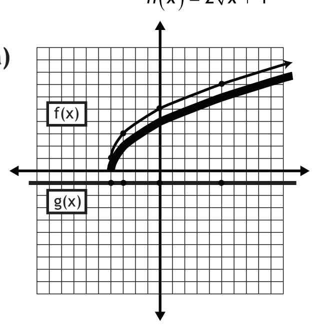
d)



Domain: $x > -2$ or (-2, ∞)
Range: $y > 0$ or (0, ∞)

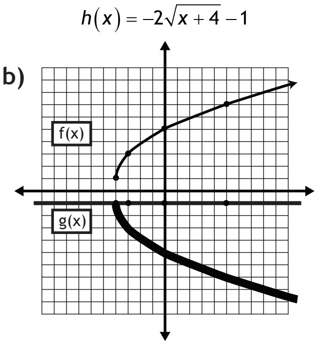
Example 4:

a) $h(x) = 2\sqrt{x+4}$



Domain: $x \ge -4$ or [-4, ∞)
Range: $y \ge 0$ or [0, ∞)
Transformation: y = f(x) - 1

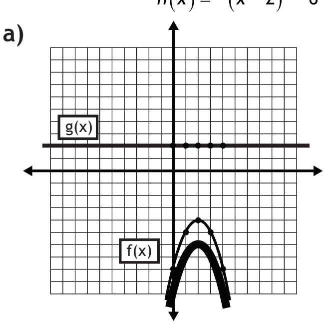
b) $h(x) = -2\sqrt{x+4} - 1$



Domain: $x \ge -4$ or [-4, ∞)
Range: $y \le -1$ or (-∞, -1]
Transformation: y = -f(x).

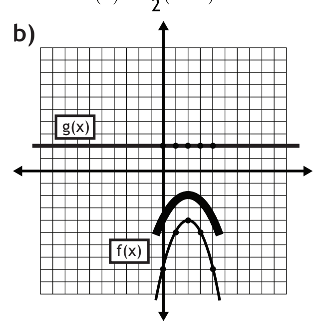
Example 5:

a) $h(x) = -(x-2)^2 - 6$



Domain: x ε R or (-∞, ∞)
Range: $y \le -6$ or (-∞, -6]
Transformation: y = f(x) - 2

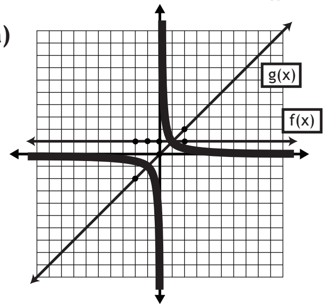
b) $h(x) = -\frac{1}{2}(x-2)^2 - 2$



Domain: x ε R or (-∞, ∞)
Range: $y \le -2$ or (-∞, -2]
Transformation: y = 1/2f(x)

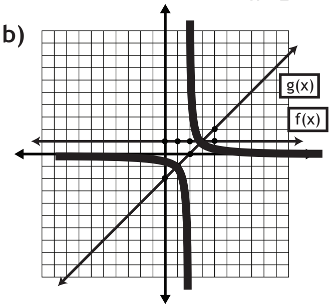
Example 6:

a) $h(x) = \frac{1}{x}$



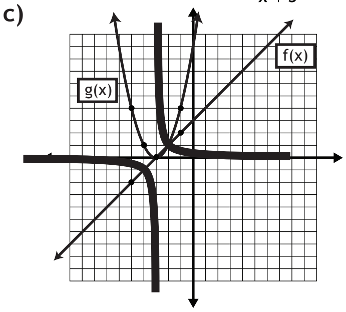
Domain: x ε R, x ≠ 0;
Range: y ε R, y ≠ 0
or D: (-∞, 0) U (0, ∞); R: (-∞, 0) U (0, ∞)

b) $h(x) = \frac{1}{x-2}$



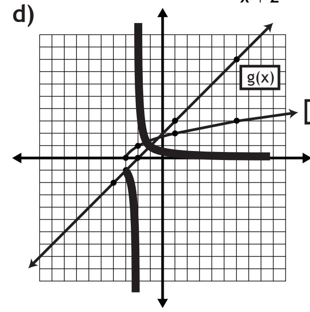
Domain: x ε R, x ≠ 2;
Range: y ε R, y ≠ 0
or D: (-∞, 2) U (2, ∞); R: (-∞, 0) U (0, ∞)

c) $h(x) = \frac{1}{x+3}$



Domain: x ε R, x ≠ -3;
Range: y ε R, y ≠ 0
or D: (-∞, -3) U (-3, ∞); R: (-∞, 0) U (0, ∞)

d) $h(x) = \frac{\sqrt{x+3}}{x+2}$



Domain: x ≥ -3, x ≠ -2;
Range: y ε R, y ≠ 0
or D: [-3, -2) U (-2, ∞); R: (-∞, 0) U (0, ∞)

Example 7:

a) $A_L(x) = 8x^2 - 8x$

b) $A_S(x) = 3x^2 - 3x$

c) $A_L(x) - A_S(x) = 10$; x = 2

d) $A_L(2) + A_S(2) = 22$;

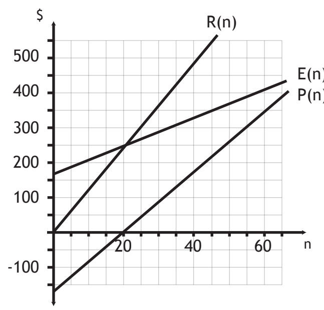
e) The large lot is 2.67 times larger than the small lot

Example 8:

a) R(n) = 12n;
E(n) = 4n + 160;
P(n) = 8n – 160

b) When 52 games are sold, the profit is $256

c) Greg will break even when he sells 20 games



Example 9:

a) The surface area and volume formulae have two variables, so they may not be written as single-variable functions.

b) $h = \sqrt{3}r$ **c)** $s = 2r$ **d)** $SA(r) = 3\pi r^2$

$$V(r) = \frac{\sqrt{3}}{3}\pi r^3$$

e) $\frac{SA}{V}(r) = \frac{3\sqrt{3}}{r}$ **f)** $\frac{SA}{V}(6) = \frac{\sqrt{3}}{2}$

Transformations and Operations Lesson Five: Function Composition

Example 1: **a)**

x	g(x)	f(g(x))
-3	9	6
-2	4	1
-1	1	-2
0	0	-3
1	1	-2
2	4	1
3	9	6

b)

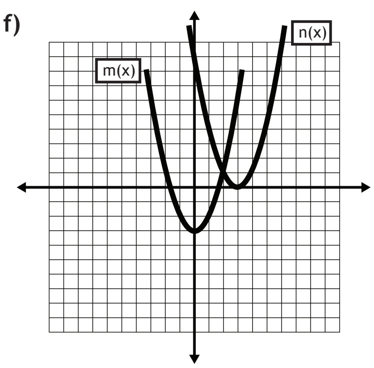
x	f(x)	g(f(x))
0	-3	9
1	-2	4
2	-1	1
3	0	0

c) Order matters in a composition of functions.

d) $m(x) = x^2 - 3$

e) $n(x) = (x - 3)^2$

f)

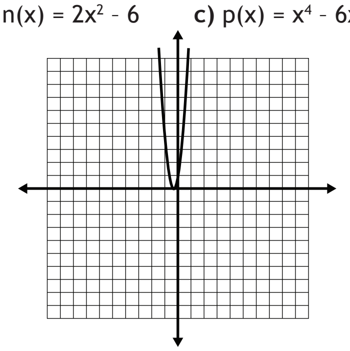


Example 2: **a)** m(3) = 33 **b)** n(1) = -4 **c)** p(2) = -2 **d)** q(-4) = -16

Example 3: **a)** $m(x) = 4x^2 - 3$ **b)** $n(x) = 2x^2 - 6$ **c)** $p(x) = x^4 - 6x^2 + 6$ **d)** q(x) = 4x **e)** All of the results match

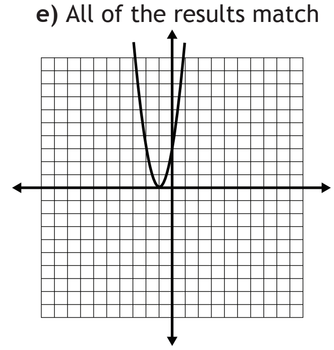
Example 4: **a)** $m(x) = (3x + 1)^2$

The graph of f(x) is horizontally stretched by a scale factor of 1/3.



b) $n(x) = 3(x + 1)^2$

The graph of f(x) is vertically stretched by a scale factor of 3.



Answer Key

Example 5: **a)** $m(x) = \sqrt{x-8}$ Domain: x ≥ 8 **b)** $m(x) = \sqrt{x-2}$ Domain: x ≥ 2

Example 6: **a)** $h(x) = \frac{1}{|x+2|}$ Domain: x ε R, x ≠ -2 **b)** $h(x) = \sqrt{x+2} + 2$ Domain: x ≥ -2

Example 7: **a)** $h(x) = \frac{1}{(x+2)^2}$ Domain: x ε R, x ≠ -2 **b)** $h(x) = \sqrt{2x+4}$ Domain: x ≥ -2

Example 8: **a)** $f(x) = 2x;\ g(x) = x+1$ **b)** $f(x) = \frac{1}{x};\ g(x) = x^2 - 1$ **c)** $f(x) = x^2 - 5x + 1;\ g(x) = x+1$

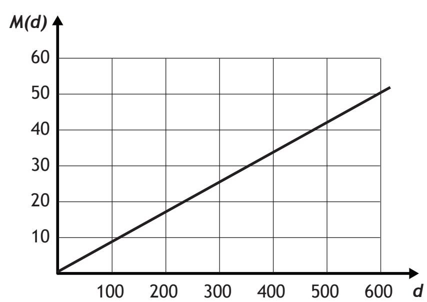
d) $f(x) = x^2;\ g(x) = x+2$ **e)** $f(x) = 2\sqrt{x};\ g(x) = \frac{1}{x}$ **f)** $f(x) = \sqrt{x};\ g(x) = x^2$

Example 9:

a) $(f^{-1} \circ f)(x) = x$, so the functions are inverses of each other.
b) $(f^{-1} \circ f)(x) \neq x$, so the functions are NOT inverses of each other.

Example 10:

a) The cost of the trip is $4.20. It took two separate calculations to find the answer.
b) V(d) = 0.08d
c) M(V) = 1.05V
d) M(d) = 0.084d
e) Using function composition, we were able to solve the problem with one calculation instead of two.



Example 11:

a) $A(t) = 900\pi t^2$

b) $A = 8100\pi\ cm^2$

c) t = 7 s; r = 210 cm

Example 12:

a) a(c) = 1.03c

b) j(a) = 78.0472a

c) b(a) = 0.6478a

d) b(c) = 0.6672c

Example 13:

a) $r(h) = \frac{3h}{8}$

b) $V_{water}(h) = \frac{3}{64}\pi h^3$

c) h = 4 cm