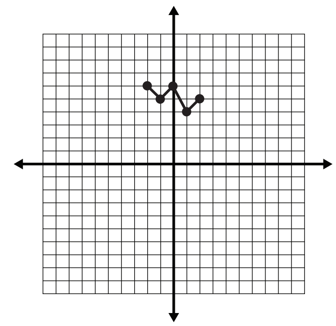
Math 30-1: Transformations and Operations
PRACTICE EXAM

1. If the graph of f(x) undergoes the transformation $y = f(\frac{1}{5}x)$, a point that exists on the graph of the image is:



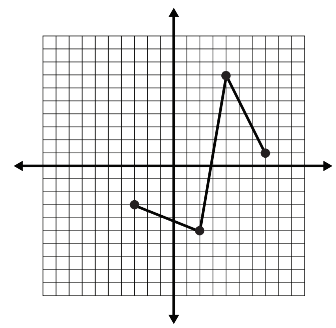
A. $\left(\frac{1}{5}, 4\right)$

B. (2, 1)

C. (-5, 5)

D. (6, 0)

2. If the graph of f(x) undergoes the transformation x = f(y), an invariant point is:



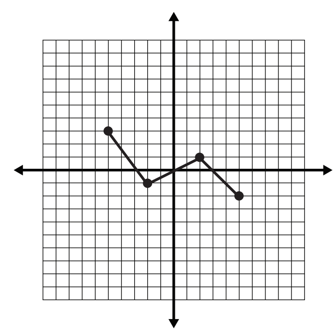
A. (7, 1)

B. (3, -3)

C. (5, 5)

D. (3, 1)

3. If the graph of f(x) undergoes the transformation y - 4 = f(x), then the range of the image is:



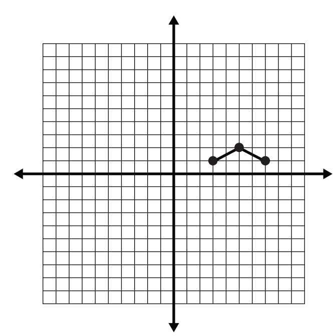
A. {y | -6 ≤ y ≤ -1, y ε R}

B. {y | 2 ≤ y ≤ 7, y ε R}

C. [-6, -1]

D. (2, 7)

4. If the graph of f(x) is horizontally translated 6 units left, then the corresponding transformation equation and mapping are:



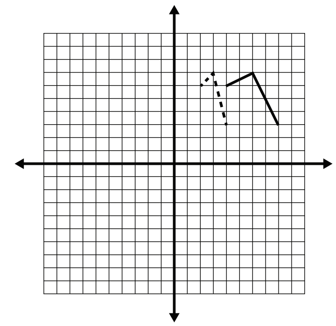
A. Transformation Equation: y = f(x - 6);
Mapping: $(x, y) \rightarrow (x - 6, y)$

B. Transformation Equation: y = f(x - 6);
Mapping: $(x, y) \rightarrow (x + 6, y)$

C. Transformation Equation: y = f(x + 6);
Mapping: $(x, y) \rightarrow (x - 6, y)$

D. Transformation Equation: y = f(x + 6);
Mapping: $(x, y) \rightarrow (x + 6, y)$

5. If f(x) *(dashed line ---)* is transformed to the image *(solid line —)*, then the corresponding transformation equation and mapping are:



A. Transformation Equation: $y = f\left(\frac{1}{2}x\right)$;
Mapping: $(x, y) \rightarrow (2x, y)$

B. Transformation Equation: $y = f\left(\frac{1}{2}x\right)$;
Mapping: $(x, y) \rightarrow \left(\frac{1}{2}x, y\right)$

C. Transformation Equation: $y = f(2x)$;
Mapping: $(x, y) \rightarrow (2x, y)$

D. Transformation Equation: $y = f(2x)$;
Mapping: $(x, y) \rightarrow \left(\frac{1}{2}x, y\right)$

6. If the graph of $f(x) = x^2 + 1$ is transformed by $g(x) = f(2x)$, then the function of the image is:

A. $g(x) = 4x^2 + 1$

B. $g(x) = 2x^2 + 1$

C. $g(x) = 2x^2 + 2$

D. $g(x) = 2x + 1$

7. If the graph of $f(x) = x^2 - 4$ is transformed by $g(x) = f(x) - 4$, then the function of the image is:

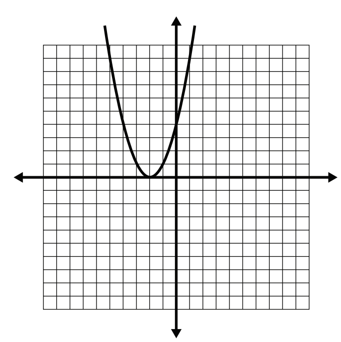
A. $g(x) = x^2 - 8$

B. $g(x) = x^2$

C. $g(x) = (x - 4)^2 - 4$

D. $g(x) = (x + 4)^2 - 4$

8. If the graph of $f(x) = (x + 2)^2$ is horizontally translated so it passes through the point (6, 9), the transformation equation is:



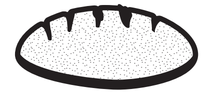
A. $y = f(x - 5)$

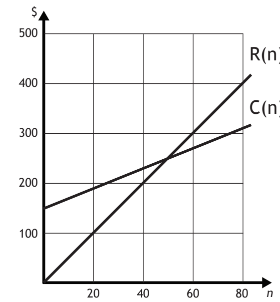
B. $y = f(x - 11)$

C. Neither $y = f(x - 5)$ nor $y = f(x - 11)$.

D. Both $y = f(x - 5)$ and $y = f(x - 11)$.

9. Sam sells bread at a farmers' market for $5.00 per loaf. It costs $150 to rent a table for one day at the farmers' market, and each loaf of bread costs $2.00 to produce. The cost (expenses) and revenue functions are:





$$C(n) = 2n + 150$$
$$R(n) = 5n$$

If the cost of renting a table increases by $50/day, and Sam raises the price of a loaf by 20%, then the new cost and revenue functions are:

A. $C_2(n) = 2n + 200$ and $R_2(n) = n$

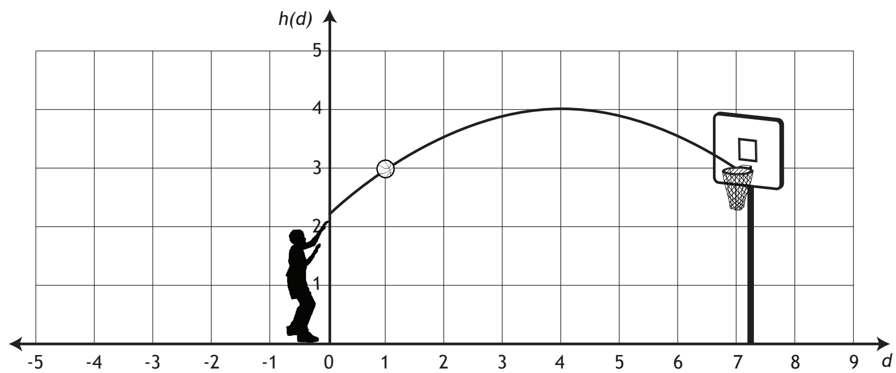
B. $C_2(n) = 2.4n + 200$ and $R_2(n) = 6n$

C. $C_2(n) = 2(n - 50) + 150$ and $R_2(n) = 5.2n$

D. $C_2(n) = 2n + 200$ and $R_2(n) = 6n$

10. A basketball player throws a basketball. The path can be modeled with the function:

$$h(d) = -\frac{1}{9}(d - 4)^2 + 4$$



If the player moves so the equation of the shot is $h(d) = -\frac{1}{9}(d + 1)^2 + 4$, the horizontal distance of the player from the hoop is:

A. 1 metre

B. 3 metres

C. 8 metres

D. 12 metres

11. The transformation $y = -3f[-4(x - 1)] + 2$ is best described (sequentially) as:

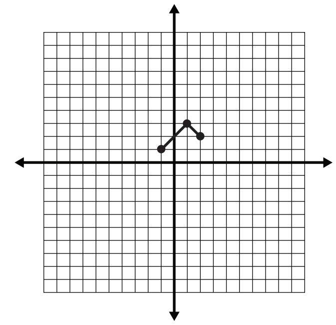
A. Translations 1 unit left and 2 units up; reflections about both the x- and y-axis; a vertical stretch by a scale factor of 3 and a horizontal stretch by a scale factor of 4.

B. Translations 1 unit right and 2 units up; reflections about both the x- and y-axis; a vertical stretch by a scale factor of 3 and a horizontal stretch by a scale factor of 1/4.

C. Reflections about both the x- and y-axis; a vertical stretch by a scale factor of 1/3 and a horizontal stretch by a scale factor of 4; and translations 1 unit right and 2 units up.

D. A vertical stretch by a scale factor of 3 and a horizontal stretch by a scale factor of 1/4; reflections about both the x- and y-axis; and translations 1 unit right and 2 units up.

12. If the graph of f(x) undergoes the transformation $y = f[\frac{1}{3}(x - 1)] + 1$, the domain and range of the image are:



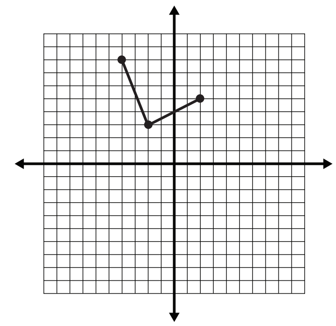
A. D: [-2, 7]; R: [2, 4]

B. D: (-2, 7); R: (2, 4)

C. D: $\{x \mid 2 \le x \le 4, x \in R\}$; R: $\{y \mid -2 \le y \le 7, y \in R\}$

D. D: $\{x \mid 2 < x < 4, x \in R\}$; R: $\{y \mid -2 < y < 7, y \in R\}$

13. If the graph of f(x) undergoes the transformation y = f(2x + 6), the horizontal translation is:



A. 2 units left.

B. 3 units left.

C. 6 units left.

D. 12 units left.

14. If the point (2, 0) exists on the graph of y = f(x), what are the coordinates of the image point after the transformation y = f(-2x + 4) is applied to the graph?

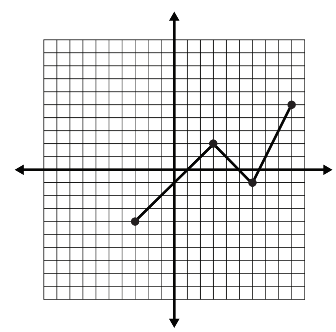
A. (-3, 0)

B. (-1, 0)

C. (0, 0)

D. (1, 0)

15. The graph of y = f(x) is horizontally stretched by a factor of $\frac{1}{3}$, reflected about the x-axis, and translated 2 units left.
The corresponding transformation equation and mapping are:



A. Transformation Equation: y = f[-3x + 2];

Mapping: $(x, y) \rightarrow \left(-\frac{1}{3}x - 2, y\right)$

B. Transformation Equation: y = -f[3x + 2];

Mapping: $(x, y) \rightarrow \left(\frac{1}{3}x - 2, -y\right)$

C. Transformation Equation: y = f[-3(x + 2)];

Mapping: $(x, y) \rightarrow \left(-\frac{1}{3}x - 2, y\right)$

D. Transformation Equation: y = -f[3(x + 2)];

Mapping: $(x, y) \rightarrow \left(\frac{1}{3}x - 2, -y\right)$

Legend for Questions 16 and 17.

VS - vertical stretch
VR - reflection about the x-axis
VT - vertical translation

HS - horizontal stretch
HR - reflection about the y-axis
HT - horizontal translation

16. The general transformation equation y = af[b(x - h)] + k can be expressed as the mapping:

$$(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$$

Based on the mapping, one can conclude that:

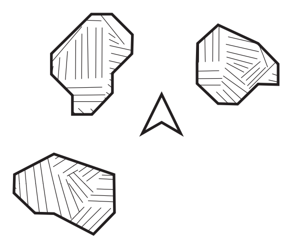
A. Transformations are axis-independent.
The transformation sequence [VS - VR - VT - HS - HR - HT] is correct because all vertical transformations are grouped together and all horizontal transformations are grouped together.

B. Stretches and reflections must universally be applied before translations.
The transformation sequence [VS - VR - VT - HS - HR - HT] is incorrect because a vertical translation is applied before a horizontal stretch.

C. Stretches and reflections can be applied in either order since the negative sign is included in the *a* and *b* parameters. The transformation sequence [VR - VS - VT - HR - HS - HT] is correct.

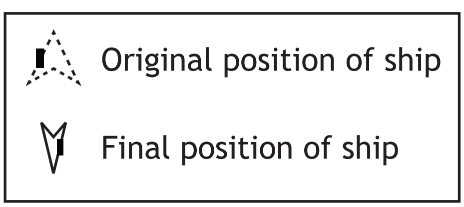
D. Both A and C are correct.

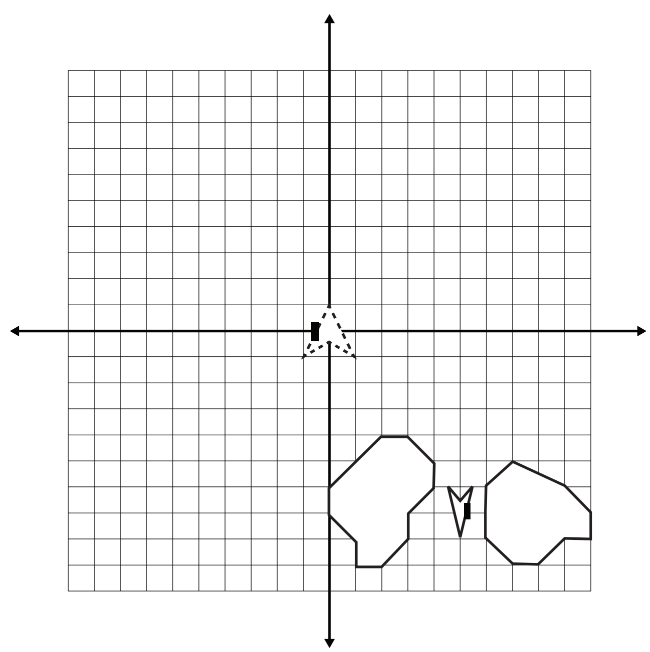
17. The goal of the video game *Space Rocks* is to pilot a spaceship through an asteroid field without colliding with any of the asteroids.



The spaceship acquires two power-ups.
The first power-up halves the original width of the spaceship, making it easier to dodge asteroids.
The second power-up is a left wing cannon.

What transformation describes the spaceship's new size and position ***and*** dodges the asteroids?





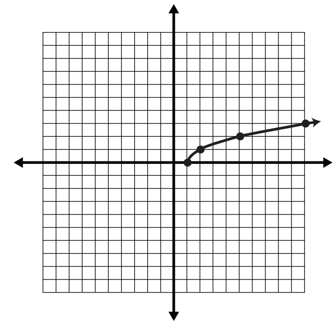
A. VR; VT = 7 down; HR; HS = 1/2; HT = 5 right

B. HS = 1/2; VR; HR; HT = 5 right; VT = 7 down

C. HT = 5 right; HR; HS = 1/2; VT = 7 down; VR

D. VT = 7 down; VR; HT = 5 right; HR; HS = 1/2

18. The graph of f(x) is shown. The domain and range of $y = f^{-1}(x)$ is:



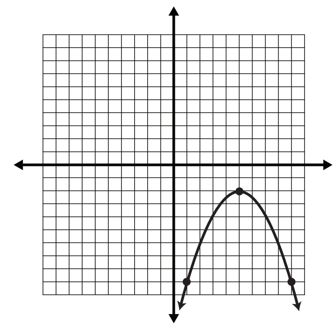
A. D: {x | x ≥ 1, x ε R}; R: {y | y ≥ 0, y ε R}

B. D: {x | x ≥ 0, x ε R}; R: {y | y ≥ 1, y ε R}

C. D: {x | x ≤ 1, x ε R}; R: {y | y ≤ 0, y ε R}

D. D: {x | x ≤ 0, x ε R}; R: {y | y ≥ 1, y ε R}

19. The graph of f(x) is shown. The graph of the inverse is a function if:



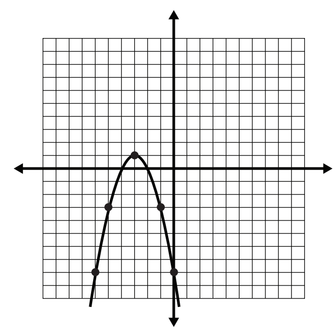
A. The shape of the inverse is a parabola opening to the left.

B. A vertical line passes through the inverse graph more than once.

C. The domain of the original graph is restricted to (-∞, 5] or [5, ∞), and then the graph is reflected about the line y = x.

D. The original graph is reflected about the line y = x.

20. The graph of $f(x) = -(x + 3)^2 + 1$ is shown. The inverse function is:



A. $x = -(y + 3)^2 + 1$

B. $f^{-1}(x) = \sqrt{-(x-1)} - 3$ only.

C. $f^{-1}(x) = -\sqrt{-(x-1)} - 3$ only.

D. $f^{-1}(x) = \sqrt{-(x-1)} - 3$ or $f^{-1}(x) = -\sqrt{-(x-1)} - 3$, but not both together.

21. If $f(x) = 2x - 6$, and $f^{-1}(k) = 18$, the value of k is:

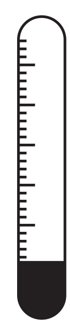
A. 12

B. 18

C. 30

D. 36

22. The formula to convert degrees Celsius to degrees Fahrenheit is $F(C) = \frac{9}{5}C + 32$. The graphs of F(C) and $F^{-1}(C)$ intersect at the point:



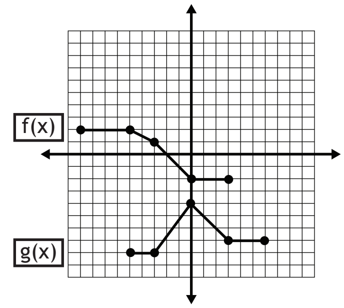
A. (-40, -40)

B. (-40, 32)

C. (32, -40)

D. (0, 32)

23. The domain of h(x) = (f - g)(x) is:



A. [-5, 3]

B. {x | -9 ≤ x ≤ 3, x ε R};

C. [-5, 6]

D. {x | -9 ≤ x ≤ 6, x ε R};

24. Given the functions f(x) = x - 3 and g(x) = -x + 1, the value of $\left(\frac{f}{g}\right)(5)$ is:

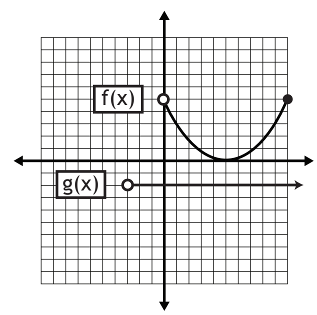
A. -2

B. $-\frac{1}{2}$

C. $\frac{1}{2}$

D. 2

25. The domain and range of h(x) = (f • g)(x) is:



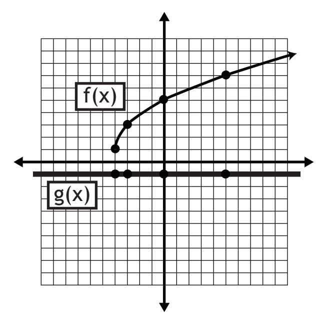
A. D: (0, 10]; R: [-10, 0]

B. D: [0, 10]; R: (-10, 0]

C. D: (0, 10]; R: (-10, 0]

D. D: (-3, 10]; R: (-10, 0]

26. Given the functions $f(x) = 2\sqrt{x + 4} + 1$ and g(x) = -1, (f • g)(x) is equivalent to the transformation:



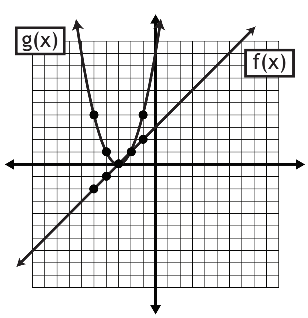
A. y = -f(x)

B. y = f(-x)

C. y = f(x) + 1

D. y = f(x) - 1

27. Given the functions f(x) = x + 3 and g(x) = x^2 + 6x + 9, the function h(x) = (f ÷ g)(x) and its domain are:



A. $h(x) = \frac{1}{x+3}$; $x \neq -3$

B. $h(x) = x + 3$; $x \neq -3$

C. $h(x) = \frac{1}{x-3}$; $x \neq 3$

D. $h(x) = x - 3$; $x \neq 3$

28. A particular cone has a height that is $\sqrt{3}$ times larger than the radius. The volume can be written as the single-variable function:

A. $V(r) = \dfrac{\sqrt{3}}{3}\pi r^3$

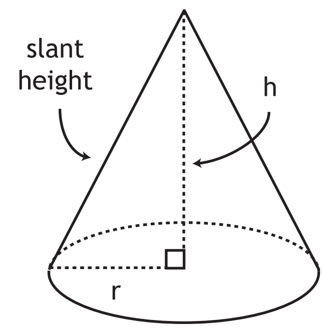
B. $V(r) = \sqrt{3}\pi r^3$

C. $V(h) = \dfrac{\sqrt{3}}{3}\pi h^3$

D. $V(h) = \sqrt{3}\pi h^3$

Cone Volume

$$V = \frac{1}{3}\pi r^2 h$$



29. Given the functions $f(x) = x^2 - 3$ and $g(x) = 2x$, the value of $(f \circ f)(2)$ is:

A. -16

B. -8

C. -4

D. -2

30. Given the functions $f(x) = x^2 - 3$ and $g(x) = 2x$, the value of $(f \circ g)(x)$ is:

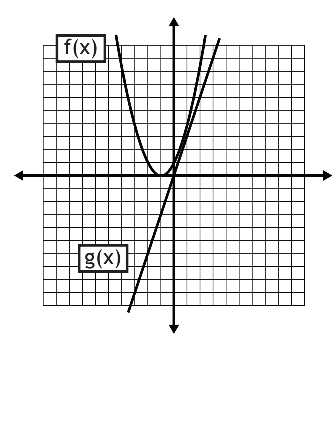
A. $2x^2 - 3$

B. $4x^2 - 3$

C. $2x^2 - 6$

D. $2x^3 - 6x$

31. Given the functions $f(x) = (x + 1)^2$ and $g(x) = 3x$, the composite function $n(x) = (g \circ f)(x)$ is equivalent to which transformation?



A. f(x) is horizontally stretched by a scale factor of three.

B. g(x) is horizontally stretched by a scale factor of three.

C. f(x) is vertically stretched by a scale factor of three.

D. g(x) is vertically stretched by a scale factor of three.

32. Given the functions $f(x) = \sqrt{x - 3}$ and $g(x) = x - 5$, the composite function m(x) = (f ∘ g)(x) is:

A. $m(x) = \sqrt{x - 8}$

B. $m(x) = \sqrt{x + 8}$

C. $m(x) = \sqrt{x - 3} - 5$

D. $m(x) = \sqrt{x + 3} - 5$

33. Given the functions f(x), g(x), m(x), and n(x), the composite function h(x) = [g ∘ m ∘ n](x) is:

$f(x) = \sqrt{x}$ $\quad g(x) = \frac{1}{x}$ $\quad m(x) = |x|$ $\quad n(x) = x + 2$

A. $h(x) = \frac{1}{|x + 2|}$

B. $h(x) = \frac{1}{|x - 2|}$

C. $h(x) = \frac{1}{|x|(x + 2)}$

D. $h(x) = x + 2$

34. Given the functions f(x), g(x), m(x), and n(x), the composite function h(x) = [f ∘ (n + n)](x) is:

$f(x) = \sqrt{x}$ $\quad g(x) = \frac{1}{x}$ $\quad m(x) = |x|$ $\quad n(x) = x + 2$

A. $h(x) = \sqrt{x + 2}$

B. $h(x) = \sqrt{2x + 4}$

C. $h(x) = \sqrt{2x - 4}$

D. $h(x) = \sqrt{2x}$

35. Given $h(x) = x^2 + 4x + 4$, where $h(x) = (f \circ g)(x)$, the functions $f(x)$ and $g(x)$ could be:

A. $f(x) = x + 2$; $g(x) = x + 2$

B. $f(x) = x - 2$; $g(x) = x - 2$

C. $f(x) = x + 2$; $g(x) = x^2$

D. $f(x) = x^2$; $g(x) = x + 2$

36. The functions $f(x) = 3x - 2$ and $g(x) = \frac{1}{3}x + \frac{2}{3}$ are inverses if:

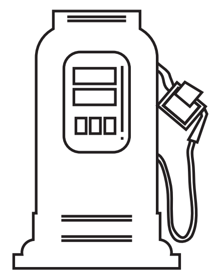
A. The graphs of $f(x)$ and $g(x)$ are symmetric about the line $y = 0$.

B. $(f \bullet g)(x) = 0$

C. $(f \circ g)(x) = 1$

D. $(f \circ g)(x) = x$

37. The price of 1 L of gasoline is \$1.05. On a level road, Darlene's car uses 0.08 L of fuel for every kilometre driven. If the volume of gas used as a function of distance is $V(d) = 0.08d$, and the money required for the trip as a function of volume is $M(V) = 1.05V$, a function that expresses the money required for the trip as a function of distance is:



A. $M(d) = 0.084d$

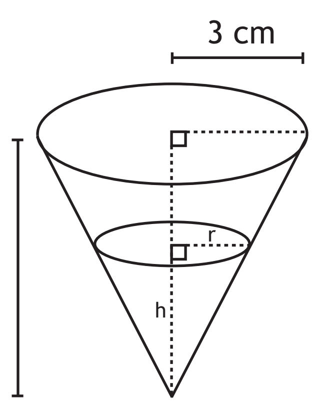
B. $M(d) = 0.08d$

C. $M(d) = 1.05d$

D. $M(V) = 1.05V$

38. A drinking cup from a water fountain has the shape of an inverted cone. The cup has a height of 8 cm and a radius of 3 cm. The water in the cup also has the shape of an inverted cone, with a radius of r and a height of h.

The volume of the cone can be written with a single variable as:



Cone Volume

$V = \frac{1}{3}\pi r^2 h$

A. $V(h) = \frac{1}{64}\pi h^3$

B. $V(h) = \frac{3}{64}\pi h^3$

C. $V(h) = \frac{1}{3}\pi h^3$

D. $V(h) = \frac{8}{3}\pi h^3$

Transformations and Operations Practice Exam - ANSWER KEY

Video solutions are in italics.

1. **C** *Basic Transformations, Example 2c*
2. **C** *Basic Transformations, Example 4c*
3. **B** *Basic Transformations, Example 6a*
4. **C** *Basic Transformations, Example 7b*
5. **A** *Basic Transformations, Example 8c*
6. **A** *Basic Transformations, Example 9b*
7. **A** *Basic Transformations, Example 10b*
8. **D** *Basic Transformations, Example 11b*
9. **D** *Basic Transformations, Example 13 (c, d)*
10. **D** *Basic Transformations, Example 14b*
11. **D** *Combined Transformations, Example 5b (iv)*
12. **A** *Combined Transformations, Example 7a*
13. **B** *Combined Transformations, Example 7b*
14. **D** *Combined Transformations, Example 8a*
15. **D** *Combined Transformations, Example 9b*
16. **D** *Combined Transformations, Example 10*
17. **B** *Combined Transformations, Example 11d*
18. **B** *Inverses, Example 2a*
19. **C** *Inverses, Example 3b*
20. **D** *Inverses, Example 5b*
21. **C** *Inverses, Example 7d*
22. **A** *Inverses, Example 8 (e,f)*
23. **A** *Function Operations, Example 1b*
24. **B** *Function Operations, Example 2d*
25. **A** *Function Operations, Example 3c*
26. **A** *Function Operations, Example 4b*
27. **A** *Function Operations, Example 6c*
28. **A** *Function Operations, Example 9d*
29. **D** *Function Composition, Example 2c*
30. **B** *Function Composition, Example 3a*
31. **C** *Function Composition, Example 4b*
32. **A** *Function Composition, Example 5a*
33. **A** *Function Composition, Example 6a*
34. **B** *Function Composition, Example 7b*
35. **D** *Function Composition, Example 8d*
36. **D** *Function Composition, Example 9a*
37. **A** *Function Composition, Example 10d*
38. **B** *Function Composition, Example 13*

Math 30-1 Practice Exam: Tips for Students

• Every question in the practice exam has already been covered in the Math 30-1 workbook. It is recommended that students refrain from looking at the practice exam until they have completed their studies for the unit.

• Do not guess on a practice exam. The practice exam is a self-diagnostic tool that can be used to identify knowledge gaps. Leave the answer blank and study the solution later.