Trigonometry

LESSON FOUR - *Trigonometric Functions II*

Lesson Notes

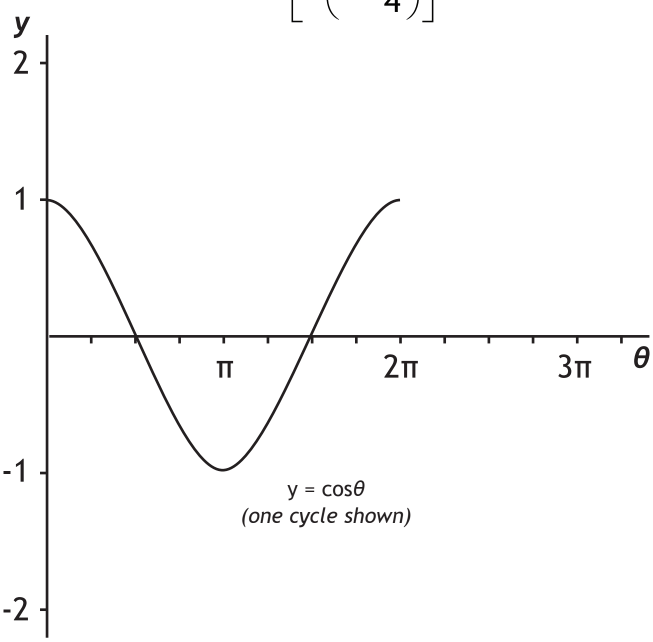
Example 1

Trigonometric Functions of Angles

Trigonometric Functions of Angles

a)

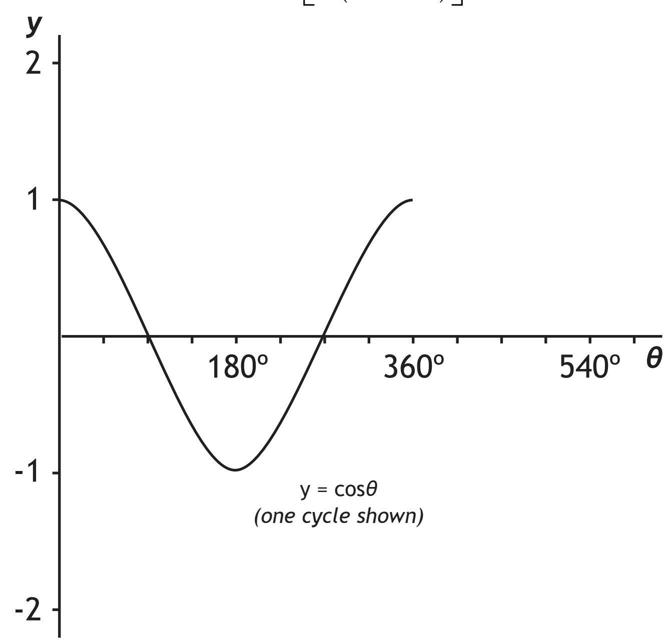
i) Graph: $f(\theta) = \cos\left[2\left(\theta - \frac{\pi}{4}\right)\right]$ $(0 \leq \theta < 3\pi)$



ii) Graph this function using technology.

b)

i) Graph: $f(\theta) = \cos\left[2\left(\theta - 45^\circ\right)\right]$ $(0^\circ \leq \theta < 540^\circ)$



ii) Graph this function using technology.

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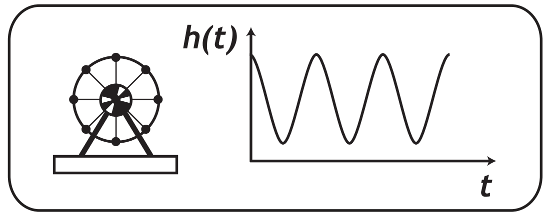
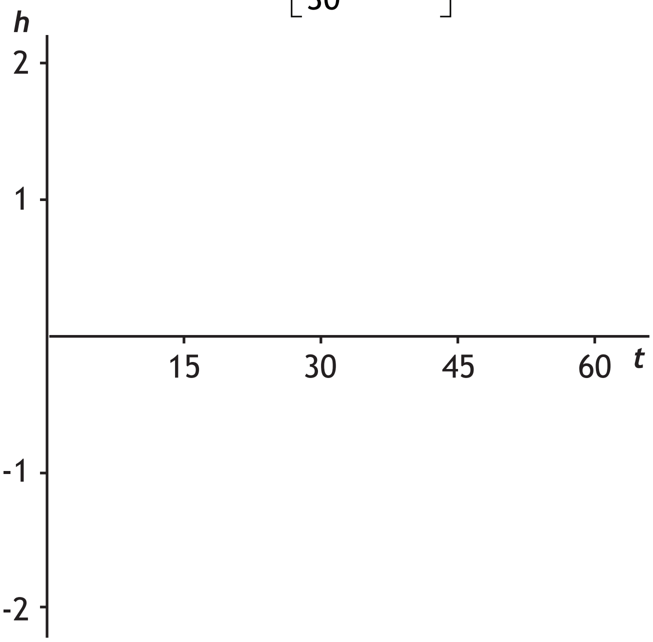


Example 2 Trigonometric Functions of Real Numbers.

Trigonometric Functions of Real Numbers

a)

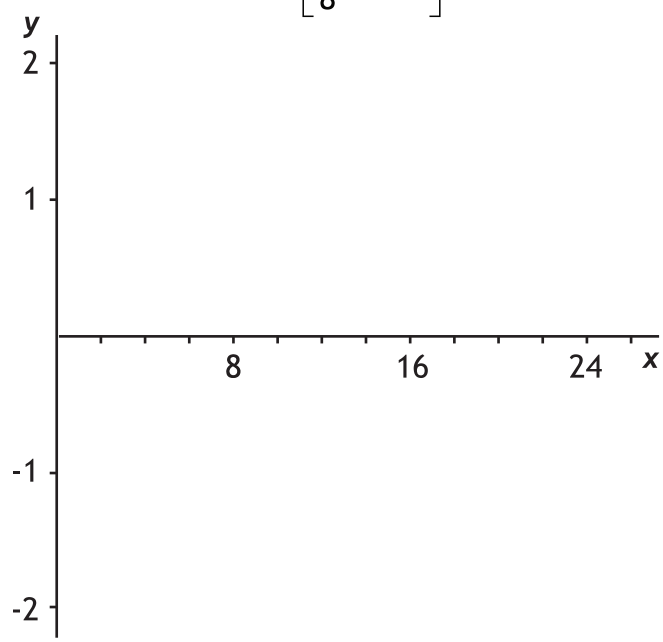
i) Graph: $h(t) = \cos\left[\frac{\pi}{30}(t - 15)\right]$



ii) Graph this function using technology.

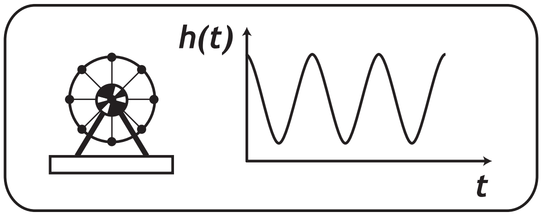
b)

i) Graph: $f(x) = \cos\left[\frac{\pi}{8}(x - 4)\right]$



ii) Graph this function using technology.

c) What are three differences between trigonometric functions of angles and trigonometric functions of real numbers?

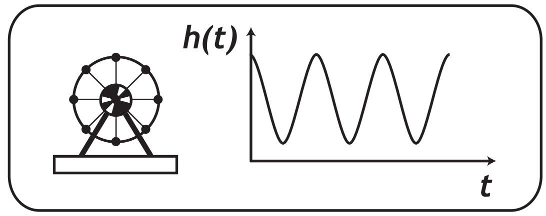


Example 3 Determine the view window for each function and sketch each graph.

Graph Preperation and View Windows

a) $f(x) = 12\sin\left[\frac{\pi}{3}(x - 2)\right] - 14$

b) $f(x) = -25\cos\frac{\pi}{250}(x + 225) + 50$



Example 4 Determine the view window for each function and sketch each graph.

Graph Preperation and View Windows

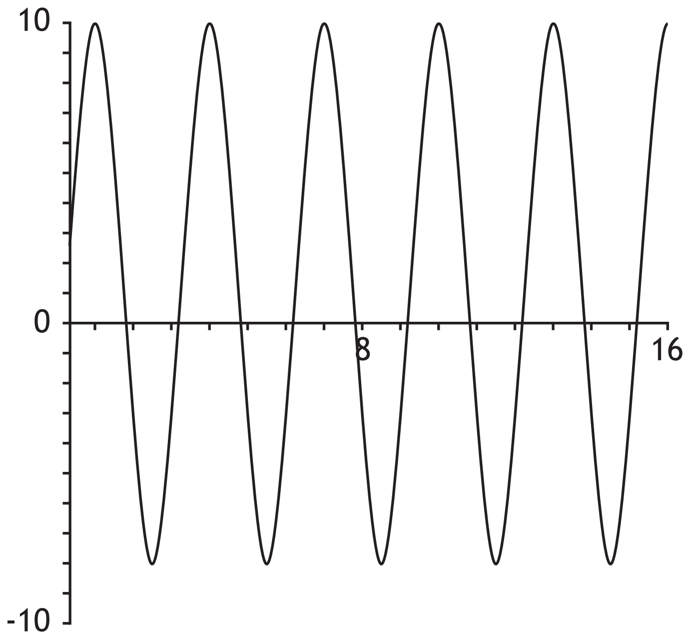
a) $f(x) = 13.5\cos\frac{2\pi}{96}(x - 24) + 6.5$

b) $f(x) = 2.5\sin 0.25\pi(x + 3) + 16$

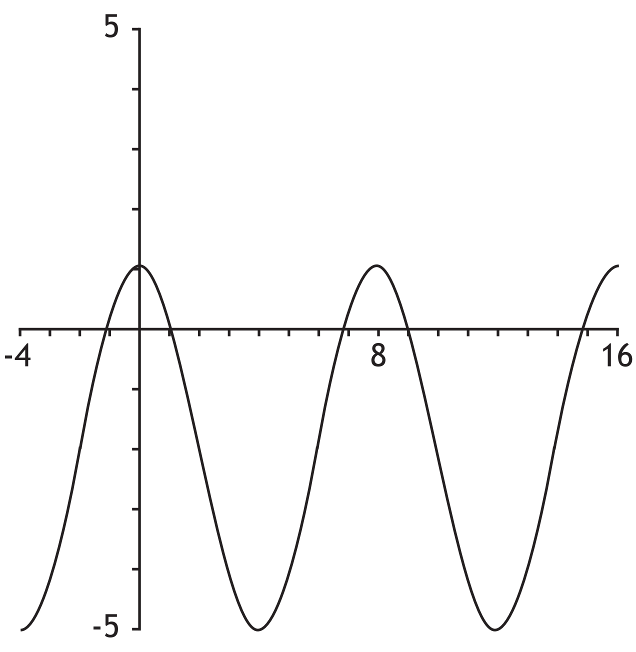
Example 5 Determine the trigonometric function corresponding to each graph.

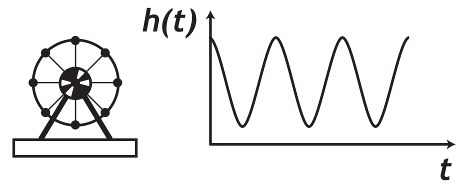
Find the Trigonometric Function of a Graph

a) write a **cosine** function.

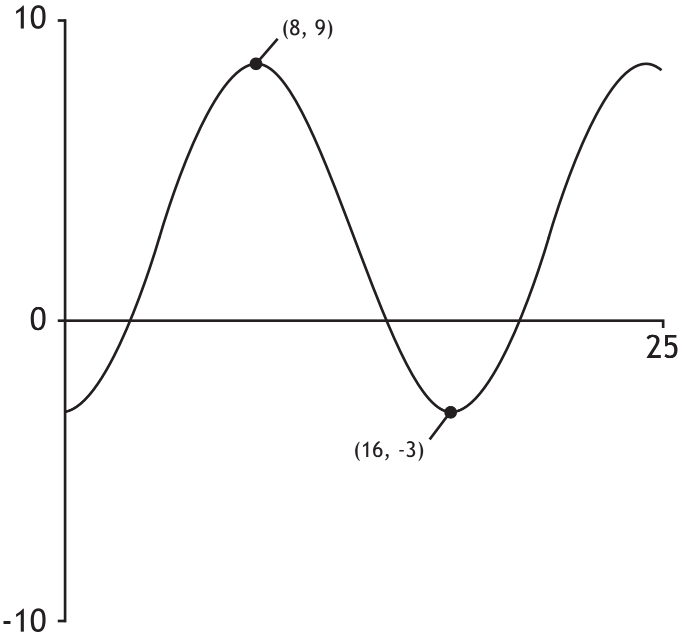


b) write a **sine** function.

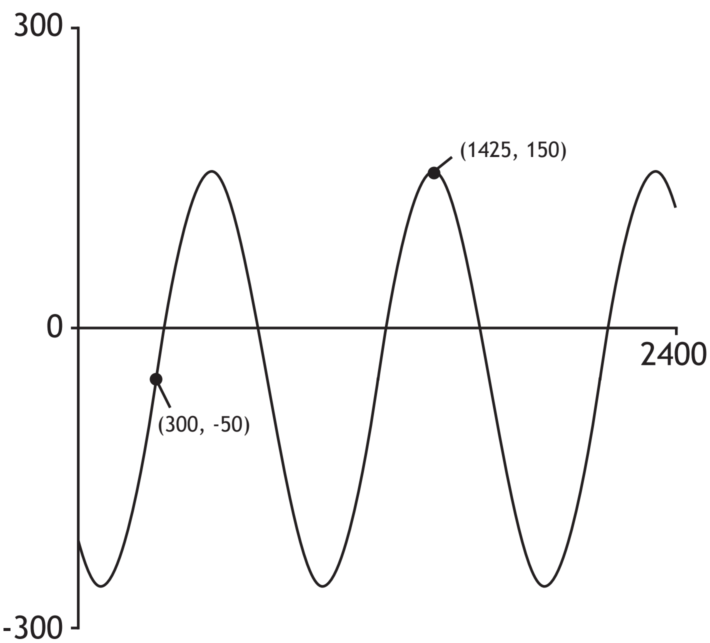


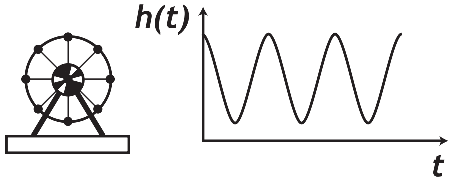


c) write a **cosine** function.



d) write a **sine** function.





Example 6 Answer the following questions:

Assorted Questions

a) If the transformation $g(\theta) - 3 = f(2\theta)$ is applied to the graph of $f(\theta) = \sin\theta$, find the new range.

b) Find the range of $f(\theta) = k\sin\left(\theta - \frac{\pi}{4}\right) - 3$.

c) If the range of $y = 3\cos\theta + d$ is $[-4, k]$, determine the values of d and k.

d) State the range of $f(\theta) - 2 = m\sin(2\theta) + n$.

e) The graphs of $f(\theta)$ and $g(\theta)$ intersect at the points $\left(\frac{\pi}{8}, \frac{\sqrt{2}}{2}\right)$ and $\left(\frac{5\pi}{8}, \frac{-\sqrt{2}}{2}\right)$

If the amplitude of each graph is quadrupled, determine the new points of intersection.

Example 7

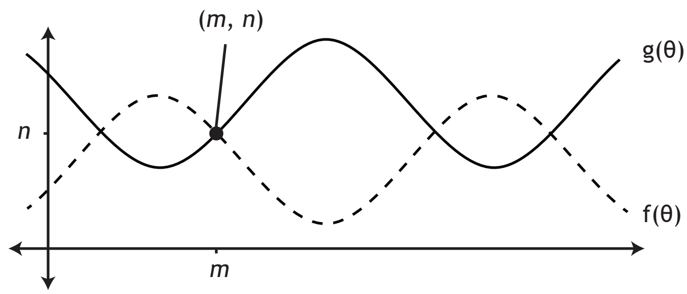
Answer the following questions:

Assorted Questions

a) If the point $\left(\frac{\pi}{2}, -2\right)$ lies on the graph of $f(\theta) = a\cos\left(\theta - \frac{\pi}{4}\right) - 4$, find the value of a.

b) Find the y-intercept of $f(\theta) = -3\cos\left(k\theta + \frac{\pi}{2}\right) - b$.

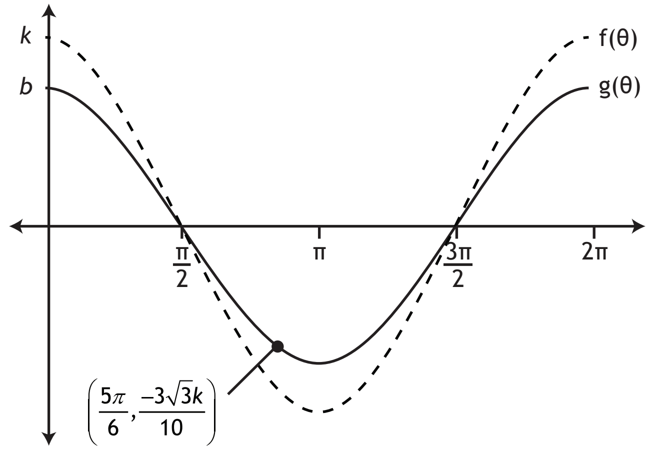
c) The graphs of f(θ) and g(θ) intersect at the point (m, n). Find the value of f(m) + g(m).



d) The graph of f(θ) = kcosθ is transformed to the graph of g(θ) = bcosθ by a vertical stretch about the x-axis.

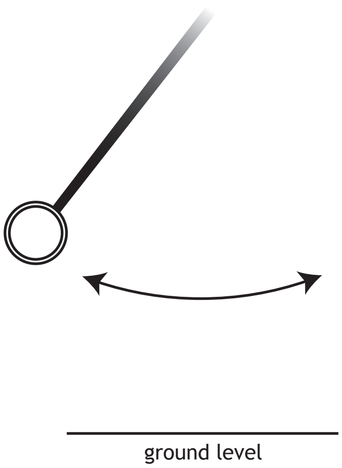
If the point $\left(\frac{5\pi}{6}, \frac{-3\sqrt{3}k}{10}\right)$ exists on the graph of g(θ),

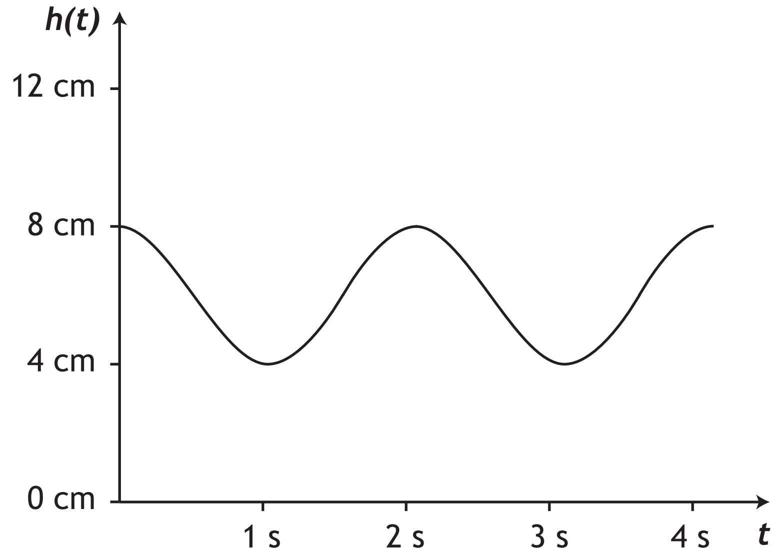
state the vertical stretch factor.



Example 8

The graph shows the height of a pendulum bob as a function of time. One cycle of a pendulum consists of two swings - a right swing and a left swing.

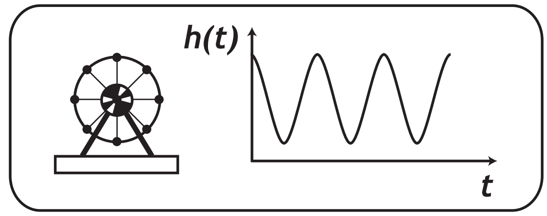




a) Write a function that describes the height of the pendulum bob as a function of time.

b) If the period of the pendulum is halved, how will this change the parameters in the function you wrote in part (a)?

c) If the pendulum is lowered so its lowest point is 2 cm above the ground, how will this change the parameters in the function you wrote in part (a)?



Example 9

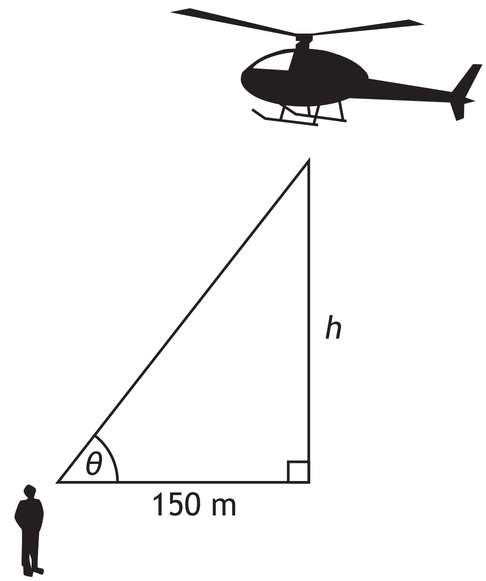
A wind turbine has blades that are 30 m long. An observer notes that one blade makes 12 complete rotations (clockwise) every minute. The highest point of the blade during the rotation is 105 m.

a) Using Point A as the starting point of the graph, draw the height of the blade over two rotations.

b) Write a function that corresponds to the graph.

c) Do we get a different graph if the wind turbine rotates counterclockwise?

Example 10

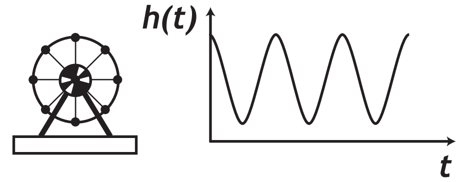


A person is watching a helicopter ascend from a distance 150 m away from the takeoff point.

a) Write a function, h(θ), that expresses the height as a function of the angle of elevation. Assume the height of the person is negligible.

b) Draw the graph, using an appropriate domain.

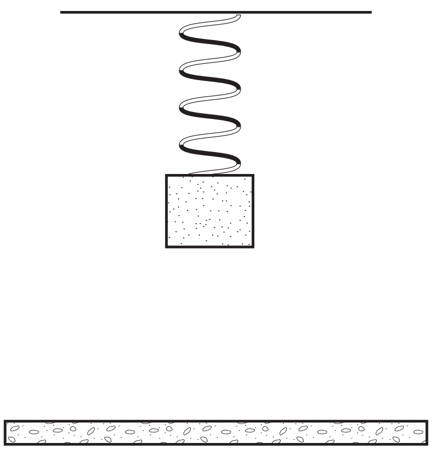
c) Explain how the shape of the graph relates to the motion of the helicopter.



Example 11

A mass is attached to a spring 4 m above the ground and allowed to oscillate from its equilibrium position. The lowest position of the mass is 2.8 m above the ground, and it takes 1 s for one complete oscillation.

a) Draw the graph for two full oscillations of the mass.



b) Write a sine function that gives the height of the mass above the ground as a function of time.

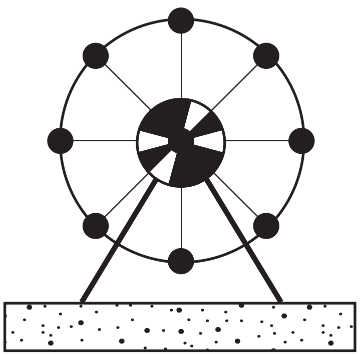
c) Calculate the height of the mass after 1.2 seconds. Round your answer to the nearest hundredth.

d) In one oscillation, how many seconds is the mass lower than 3.2 m? Round your answer to the nearest hundredth.

Example 12

A Ferris wheel with a radius of 15 m rotates once every 100 seconds. Riders board the Ferris wheel using a platform 1 m above the ground.

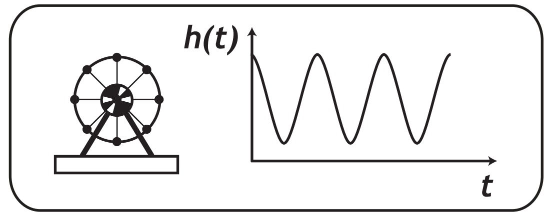
a) Draw the graph for two full rotations of the Ferris wheel.



b) Write a cosine function that gives the height of the rider as a function of time.

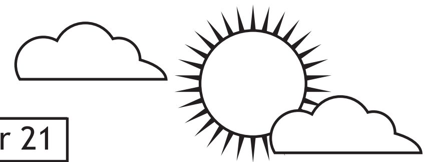
c) Calculate the height of the rider after 1.6 rotations of the Ferris wheel.
Round your answer to the nearest hundredth.

d) In one rotation, how many seconds is the rider higher than 26 m?
Round your answer to the nearest hundredth.



Example 13

The following table shows the number of daylight hours in Grande Prairie.

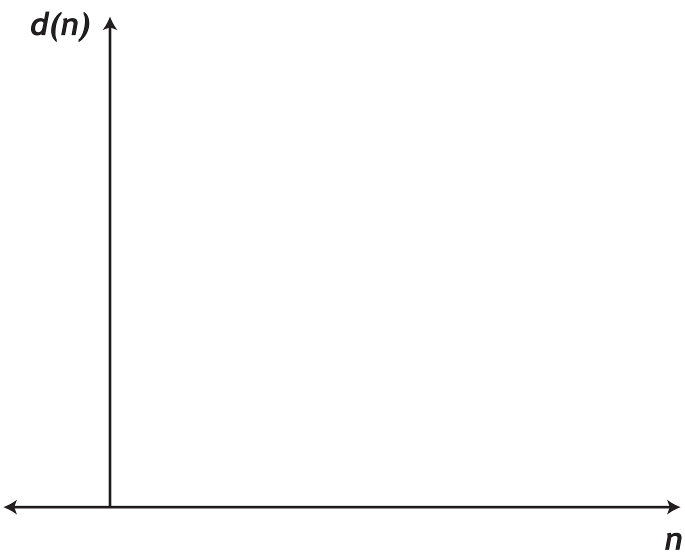


December 21	March 21	June 21	September 21	December 21
6h, 46m	12h, 17m	17h, 49m	12h, 17m	6h, 46m

a) Convert each date and time to a number that can be used for graphing.

Day Number	December 21 =	March 21 =	June 21 =	September 21 =	December 21 =
Daylight Hours	**6h, 46m =**	**12h, 17m =**	**17h, 49m =**	**12h, 17m =**	**12h, 46m =**

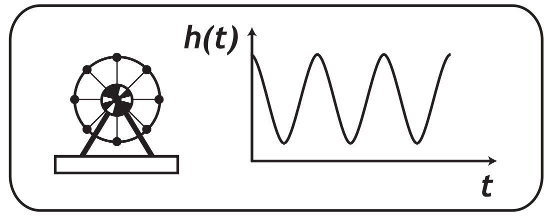
b) Draw the graph for one complete cycle *(winter solstice to winter solstice)*.



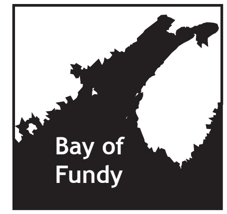
c) Write a cosine function that relates the number of daylight hours, *d*, to the day number, *n*.

d) How many daylight hours are there on May 2? Round your answer to the nearest hundredth.

e) In one year, approximately how many days have more than 17 daylight hours?
Round your answer to the nearest day.



Example 14



The highest tides in the world occur between New Brunswick and Nova Scotia, in the Bay of Fundy. Each day, there are two low tides and two high tides. The chart below contains tidal height data that was collected over a 24-hour period.

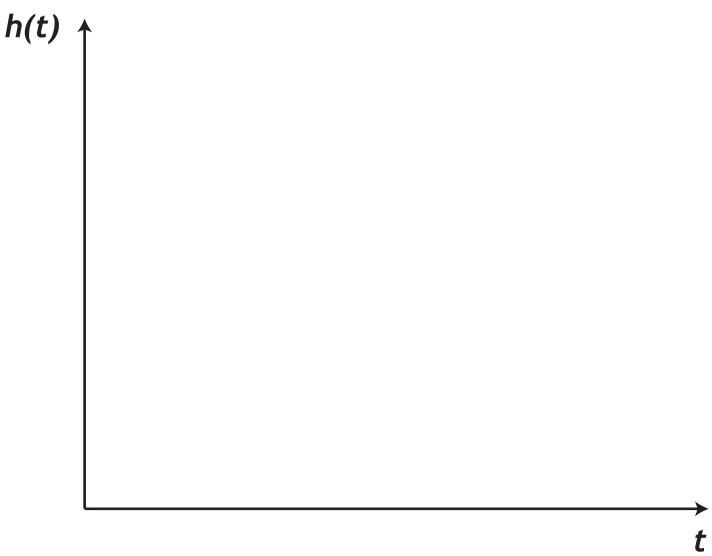
	Time	Decimal Hour	Height of Water (m)
Low Tide	2:12 AM		3.48
High Tide	8:12 AM		13.32
Low Tide	2:12 PM		3.48
High Tide	8:12 PM		13.32

Note: Actual tides at the Bay of Fundy are 6 hours and 13 minutes apart due to daily changes in the position of the moon.

In this example, we will use 6 hours for simplicity.

a) Convert each time to a decimal hour.

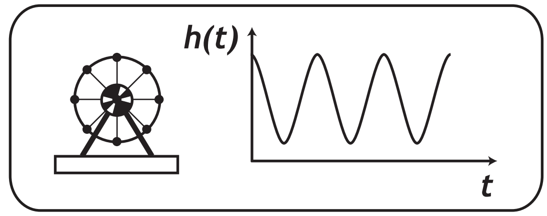
b) Graph the height of the tide for one full cycle *(low tide to low tide)*.



c) Write a cosine function that relates the height of the water to the elapsed time.

d) What is the height of the water at 6:09 AM? Round your answer to the nearest hundredth.

e) For what percentage of the day is the height of the water greater than 11 m?
Round your answer to the nearest tenth.



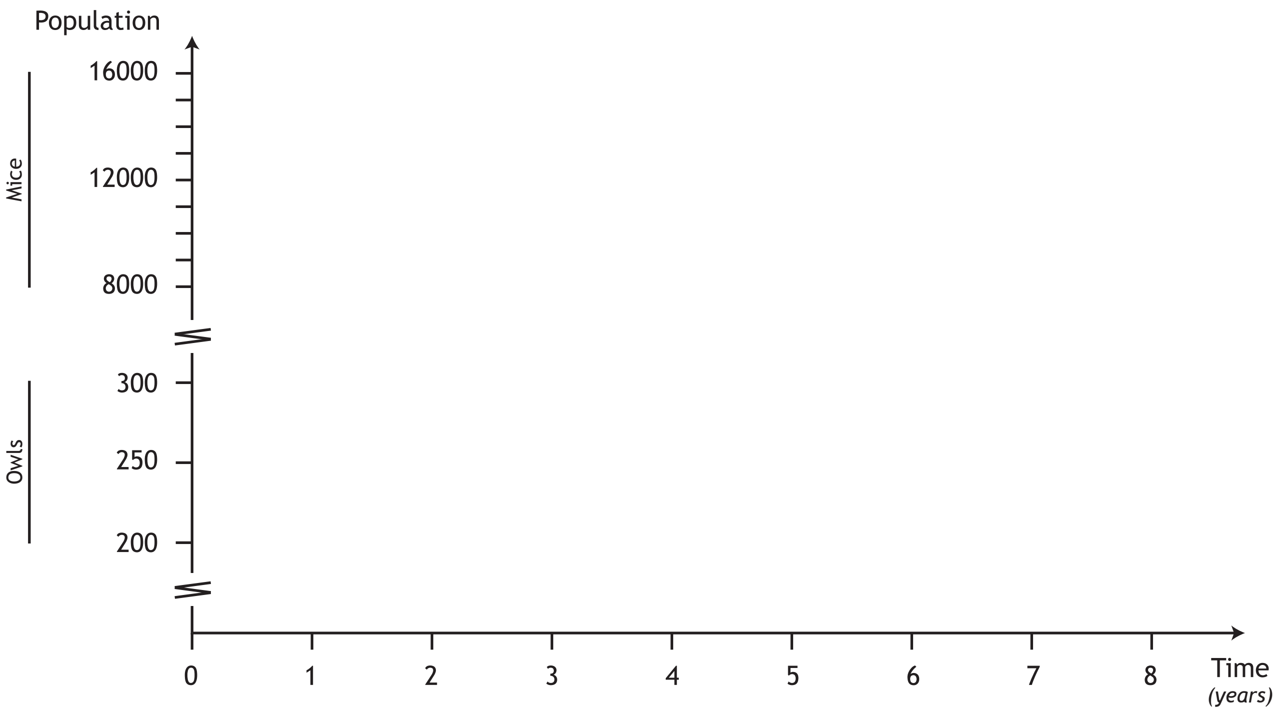
Example 15

A wooded region has an ecosystem that supports both owls and mice. Owl and mice populations vary over time according to the equations:

Owl population:	$O(t) = 50\sin\left[\frac{\pi}{3}(t-1.5)\right] + 250$
Mouse population:	$M(t) = 4000\sin\left(\frac{\pi}{3}t\right) + 12000$

where O is the population of owls, M is the population of mice, and t is the time in years.

a) Graph the population of owls and mice over six years.



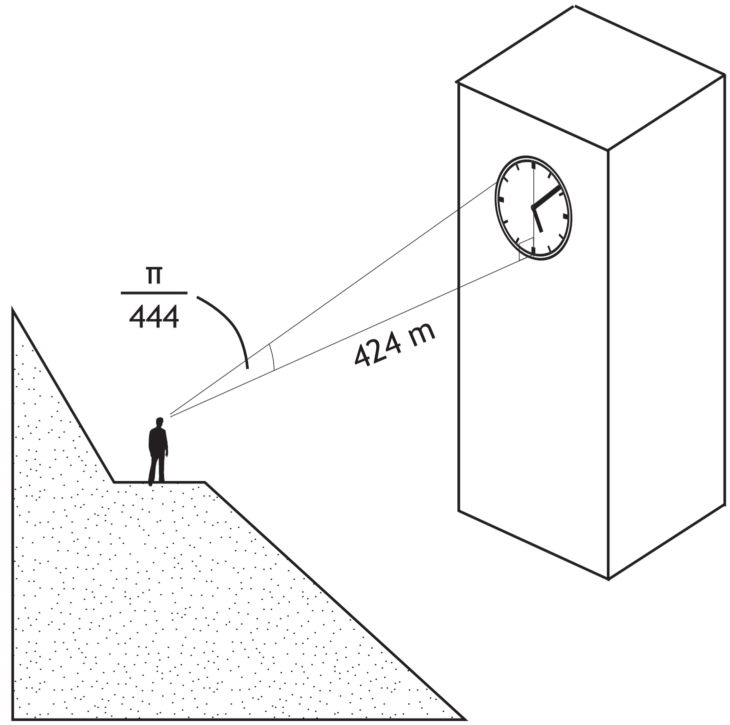
b) Describe how the graph shows the relationship between owl and mouse populations.

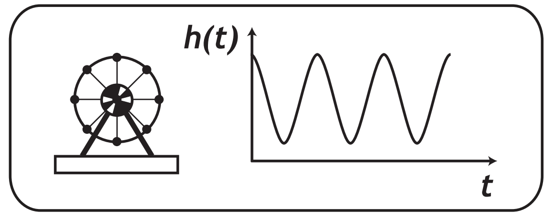
Example 16

The angle of elevation between the 6:00 position and the 12:00 position of a historical building's clock, as measured from an observer standing on a hill, is $\frac{\pi}{444}$.

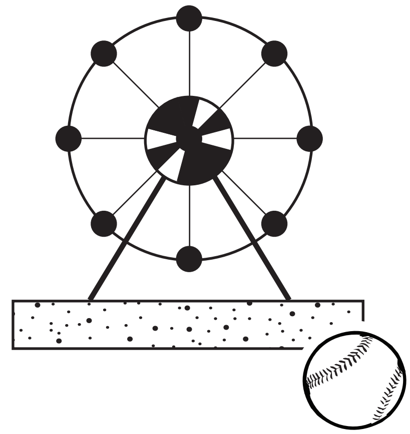
The observer also knows that he is standing 424 m away from the clock, and his eyes are at the same height as the base of the clock. The radius of the clock is the same as the length of the minute hand.

If the height of the minute hand's tip is measured relative to the bottom of the clock, what is the height of the tip at 5:08, to the nearest tenth of a metre?





Example 17



Shane is on a Ferris wheel, and his height can be described by the equation $h_{wheel}(t) = -9\cos\frac{\pi}{30}t + 10$.

Tim, a baseball player, can throw a baseball with a speed of 20 m/s. If Tim throws a ball directly upwards, the height can be determined by the equation
$h_{ball}(t) = -4.905t^2 + 20t + 1$

If Tim throws the baseball 15 seconds after the ride begins, when are Shane and the ball at the same height?